

UNIVERSITY OF WARSAW
FACULTY OF ECONOMIC SCIENCES

PHD DISSERTATION

Selected Issues on the Optimal Endogenous Growth Policies

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February, 2018

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Chapter 1

Introduction

Three notions have lately come to the center of attention in macroeconomics. First, the behavior of research and development expenditures is highly procyclical, which may have significant consequences for growth in face of severe shocks (see Figure 1.1). Second, the cyclical behavior of establishments (plants) is of utmost importance to growth in the long run, while exhibiting very turbulent patterns over the business cycle (see Figure 1.3). Third, changes in entry and exit patterns of establishments (see Figure 1.2) have significant consequences on the behavior of other macroeconomic variables.

My original contribution to knowledge is twofold. First, I build a tractable model of heterogeneous, monopolistically competitive establishments that endogenously choose the intensity of research and development activities, and are subject to endogenous entry and exit pressures. Both entry and exit pressures, as well as incentives for conducting research and development vary over the business cycle, generating procyclical pattern of aggregate research and development expenditures. Second, I apply this model to investigate how certain economic policies, specifically static and countercyclical subsidies, affect incentives for conducting research and development activities and how this translates to the behavior of other macroeconomic variables and what are their welfare effects.

The key equation that is responsible for the majority of results obtained in this thesis, and is derived formally in Chapter 2, can be informally expressed as follows:

$$\text{endogenous growth rate} = \text{contribution of incumbents} \times \text{contribution of entrants} \quad (1.1)$$

The rate of endogenous growth depends on the contribution of two groups of economic agents: already active establishments (incumbents), and new establishments (entrants). The key reason why this equation is of interest is that the efforts of both incumbents and (potential) entrants depend on the phase of the business cycle. As a consequence, business cycle fluctuations affect the underlying rate of economic growth, and can have long-lasting effects. The objective of this thesis is to integrate several strands of literature, including endogenous growth, business cycle and firm and establishment dynamics literature, and provide a new assessment of the welfare cost of business cycles, as well as novel guidance on industrial policy.

I find that business cycle fluctuations have a noticeable impact on the endogenous growth rates. The results from the baseline model, as well as its extensions, indicate that around 6-7% of a transitory shock to Total Factor Productivity (TFP) is translated to a permanent shift in the Balanced Growth Path (BGP) of the model economy, and

around 50% of this effect is already in place after 5 years following the initial shock. This generates significant welfare consequences, as the resulting cost of business cycle fluctuations is of two orders of magnitude higher than in the exogenous growth variant of the model.

The presence of large welfare effects and the ability to potentially affect the growth rates and volatility of the economy through appropriate industrial policy creates ample space for policy intervention. For example, positive welfare effect is achieved through countercyclical subsidies to incumbents' operating cost, as it prevents excessive exits and encourages more R&D spending. Moreover, once the costs of labor market turnover over the business cycle are appropriately accounted for, it turns out that static subsidies to incumbents' operating cost are also welfare improving, a result at odds with prior findings in the endogenous growth literature, which did not pay much attention to the effects of business cycle fluctuations.

The dissertation is structured as follows. The remainder of this Chapter provides a review of the relevant literature, both in the areas of endogenous growth and business cycles, and documents several stylized facts on research and development expenditures and establishment dynamics over the business cycle. Chapter 2 develops the baseline model, presents the method for solving for both the Balanced Growth Path (BGP) and the stochastic equilibrium, and then applies it to study the effects a number of static and countercyclical subsidy schemes. Chapter 3 extends the baseline model by including physical capital and frictional labor markets, which allows to examine the effects of subsidy schemes in a richer context. In Chapter 4 the model is further extended by including financial shocks, which allows me to capture salient features of the 2008-2009 financial crisis and its aftermath. The last Chapter concludes and discusses the policy recommendations stemming from the results of the previous Chapters.

1.1 Related literature and stylized facts on R&D expenditures and establishment dynamics

Research in macroeconomics is predominantly focused on two broad topics: long-run growth and business cycles, which used to be separate topics analyzed with markedly different models and tools. Thanks to the seminal contributions of Ramsey (1928) and Solow (1956) in the field of economic growth, and Brock & Mirman (1972) and Kydland & Prescott (1982) in the business cycles research, the (stochastic) neoclassical growth model has become the workhorse model and vantage point in both strands of macroeconomic literature. This unification has led Cooley & Prescott (1995) to proclaim that “growth and fluctuations are not distinct phenomena to be studied with separate data and different analytical tools”. Despite his optimism, both fields to this day remain largely separate.

The late 1980s and early 1990s have seen the emergence of the first generation of endogenous growth theory with the breaking-ground contributions by Romer (1987), Romer (1990), Grossman & Helpman (1991) and Aghion & Howitt (1992, 1998, 2008). Especially the work by Aghion and Howitt rekindled the interest in the then fifty years old ideas of creative destruction by Schumpeter (1942). Jones (1995) pointed out a fatal flaw in the first generation of endogenous growth models, namely the presence of scale effects, and in attempt to rectify the defect, he proposed a semi-endogenous growth model. The subsequent intellectual effort gave rise to the so-called second generation of endogenous growth models.

From the perspective of this dissertation, the contributions by Dinopoulos & Thompson (1998), Peretto (1998), Young (1998) and Howitt (1999) all share the interesting feature in that the attention is redirected from the entire economy toward individual establishments. In short, while in the aggregate the population of R&D scientists may rise, the important statistic is the R&D labor per establishment, which remains constant under mild assumptions regarding the market structure. Indeed, Laincz & Peretto (2006) show that since 1964 the number of full-time equivalent R&D employees per establishment has been almost constant and was not trending over time.

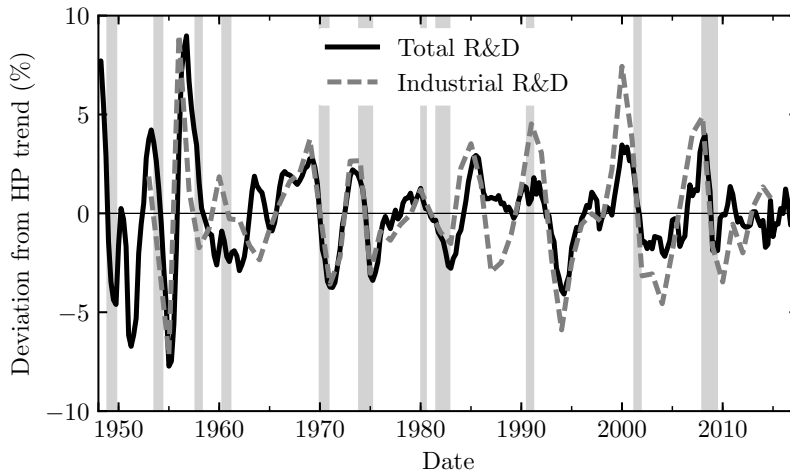
The idea that business cycles and endogenous growth are intertwined is not new. In one of the early contributions, Ozlu (1996) shows that shocks to learning-by-doing and human capital investment processes can generate aggregate fluctuations similar to business cycles. Further work in this vein of endogenous business cycles includes papers by e.g. Maliar & Maliar (2004), Jones et al. (2005), Walde (2005), Phillips & Wrase (2006), where various factors influencing endogenous growth rate are found to generate persistent business cycles. In an interesting vein of research Gabaix (2011) and Acemoglu et al. (2012) utilize the granularity and networking features of real economies to argue that in such circumstances idiosyncratic shocks do not average out and give rise to aggregate fluctuations. Rozsypal (2015) models the consequences of market complementarities and imperfect information that lead to a contagion-type effects of idiosyncratic innovative activities, causing persistent business cycles even if idiosyncratic shocks are not persistent themselves.

The wealth of works analyzing endogenous business cycles contrasts with the scarcity of literature that considers the causality going in the other direction, i.e. the impact of business cycle fluctuations on the endogenous growth rates. Among the few papers that do so, Fatas (2000) relies on aggregate demand externalities modeled after Shleifer

(1986) to generate more persistent business cycles that impact R&D expenditures and growth. Comin & Gertler (2006) employ the notion of medium-term business cycles. In their work, transitory TFP shocks procyclically influence invention of new technologies and adoption of existing ones, creating more persistent effects¹. Anzoategui et al. (2016) successfully extend this framework to argue that large demand shocks at the onset of the Great Recession and the subsequent drop in R&D activity may explain the weak recovery.

As argued by Barlevy (2004), if business cycles influence endogenous growth rate of the economy, the welfare cost of business cycles as estimated by Lucas (1987) may be biased downward by a few orders of magnitude. Fatas (2000) and Barlevy (2007) provide ample evidence that R&D expenditures in the US are volatile and procyclical², as can be seen in Figure 1.1. However, as Aghion & Saint-Paul (1998) point out, under standard endogenous growth framework a rational firm manager would find it optimal to engage in R&D more intensively during recessions, as the opportunity cost of R&D relative to production drops, generating countercyclical R&D expenditure patterns. Therefore, Barlevy (2007) relies on dynamic externalities to R&D making entrepreneurs “short-sighted” to counteract the aforementioned effect. Nuño (2011) extends the model of Aghion & Howitt (1998) to a stochastic setting and is able to generate procyclical R&D pattern, although his model does not allow for innovation by incumbents.

Figure 1.1: Cyclical behavior of US R&D expenditures, 1948q1-2017q3



Source: Total R&D: US Bureau of Economic Analysis (BEA), Gross Domestic Product: Research and Development. Industrial R&D: National Science Foundation (NSF), Industrial R&D performed in the United States, by source of funds, expenditures financed from all private and non-federal sources. The second time series is available at annual frequency for years 1953-2015. Deflated by the GDP deflator and detrended with the Hodrick & Prescott (1997) (HP) filter. Shaded bars indicate NBER recessions.

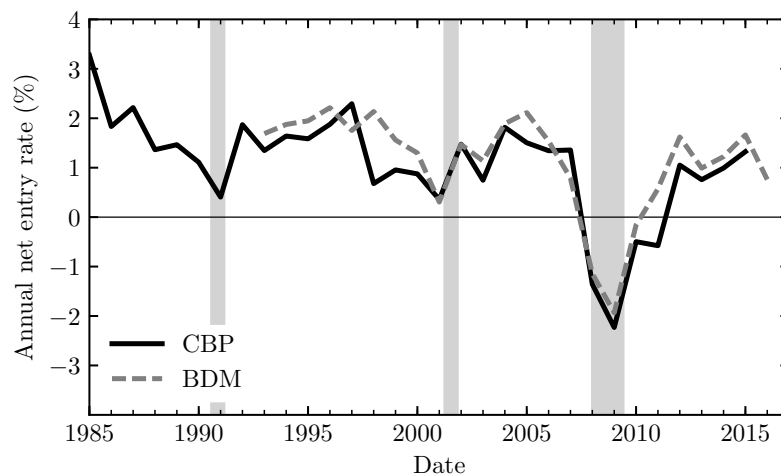
While procyclicality of R&D expenditures may be the main factor affecting long-run endogenous growth, impacting welfare via shifts in the Balanced Growth Path (BGP),

¹The key difference between Comin & Gertler (2006) and my work is that they use an ad-hoc, rather than microfounded aggregate innovation functions. This obviously precludes analysis of industrial policy.

²Walde & Woitek (2004) find similar patterns among other G-7 countries for the 1973-2000 period.

another source of welfare consequences of business cycles are firm and establishment dynamics. The early seminal contributions in the field of industry equilibrium and dynamics are Jovanovic (1982) and Hopenhayn (1992). Both of these papers assume that a firm's productivity is drawn from a certain distribution at entry, and remains fixed afterwards. Bartelsman & Doms (2000) provide a review of the early literature focused on documenting productivity differences and growth across firms and linking these phenomena to aggregate outcomes. Foster et al. (2001) emphasize the role of cyclical entry for aggregate productivity growth. The role of entry and exit channels for macroeconomic dynamics has been recognized and studied by e.g. Devereux et al. (1996), Campbell (1998), Jaimovich & Floetotto (2008), Samaniego (2008), Bilbiie et al. (2012), Chatterjee & Cooper (2014) and Lee & Mukoyama (2015), although none of these works incorporate the full set of firm dynamics considered here.

Figure 1.2: Cyclical behavior of US establishment annual net entry rates, 1985-2016

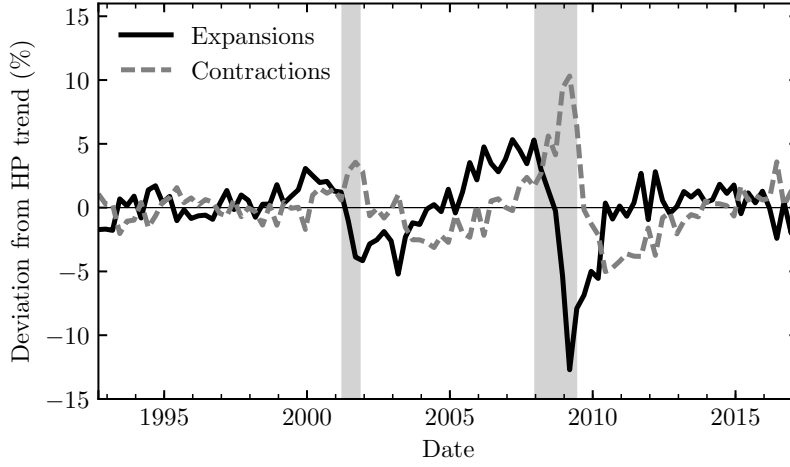


Source: CBP: US Census Bureau, County Business Patterns. BDM: US Bureau of Labor Statistics (BLS), Business Employment Dynamics. The second time series is available starting in 1992q3.

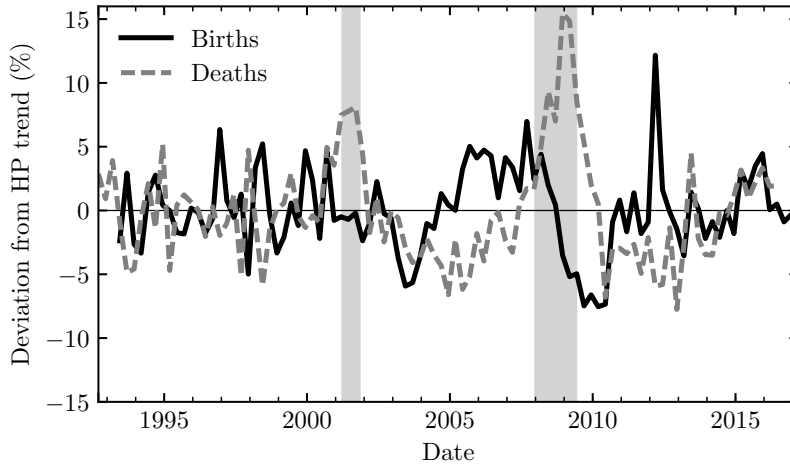
Note: the definition of net entry from BDM is (births-deaths) over the average number of establishments in two consecutive periods. Average annual net entry rate is slightly above 1%.

Recently, the experience of the Great Recession and the subsequent slow recovery motivated researchers to investigate possible links between cyclical changes in firm and establishment dynamics and other macroeconomic variables, a phenomenon dubbed missing generation of firms. Clementi & Palazzo (2016) study full firm dynamics over business cycle, although their analysis focuses on the firm-level investment in physical capital, rather than innovation. Messer et al. (2016) show using regional US data that low entry rates in the 2007-2009 period contributed significantly to low employment and labor productivity growth. Siemer (2014) finds that tight financial constraints during the Great Recession were responsible for both low employment growth and firm entry rates. Indeed, a cursory look at establishment dynamics in Figures 1.2 and 1.3 reveals a very turbulent picture, with significant deviations from the trend during the dot-com bubble recession of 2001 and very large deviations during the financial crisis of 2008-2009.

Figure 1.3: Cyclical behavior of US establishments, 1992q3-2017q1



(a) Expansions and Contractions



(b) Births (entries) and Deaths (exits)

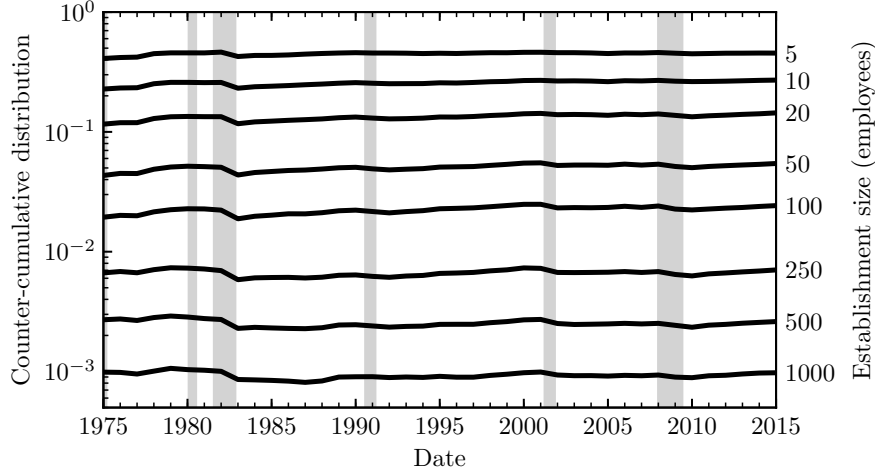
Source: US Bureau of Labor Statistics, Business Employment Dynamics. Detrended with HP filter.

In the light of above facts, a surprising feature of the US economy is the remarkably stable distribution of establishment sizes (measured in number of employees), a fact emphasized by Rossi-Hansberg & Wright (2007) and Luttmer (2010). Figure 1.4 tracks this distribution over time, with the left hand axis showing the proportion of establishments employing at least as many employees as the respective cutoff points on the right hand size axis indicate. The distribution exhibits very little, if any, cyclical variation. The biggest visible change in the size distribution occurs between years 1982-1983, although it is not directly related to the period's recession, but rather to a change in the statistical definition of an active establishment.³ Therefore, any theory describing cyclical behavior

³A methodological note of the County Business Patterns reads: "This series represents an extension of a program that has been published annually since 1964 and at irregular intervals dating back to

of establishments has to be consistent with the apparent time and cycle invariance of their size distribution. One approach to guarantee it is to assume or generate a distribution of sizes that follows a power law distribution, e.g. Pareto, as these distributions are invariant with respect to their left cutoff point.

Figure 1.4: Establishment size distribution over time, 1975-2015



Source: US Census Bureau, County Business Patterns. The figure plots the counter-cumulative distribution of establishments' size in terms of employment. The numbers at the right hand size axis denote the size cutoff points, e.g. the curve corresponding to number 5 tracks the proportion of establishments that employ at least 5 employees.

Following the seminal contribution by Klette & Kortum (2004), there is a frugal literature on the relationship between innovation and firm dynamics. Klette & Kortum (2004) merge endogenous growth theory with industry dynamics, with firms innovating in order to enhance their product lines portfolio and growth emerging as the result of creative destruction. Lentz & Mortensen (2008) use a panel of Danish firms and are able to provide empirical support for the model. Acemoglu et al. (2013) develop a parsimonious model where firms have either high or low innovative capacity and show that they can generate steady-state behavior consistent with the stylized facts such as high growth rates and high exit probability among young firms. Related works include Akcigit & Kerr (2010) and Acemoglu & Cao (2015). The common assumption in these papers is that the incumbent firms innovate on their own products in a neo-Schumpeterian quality-ladder setup. I contribute to that literature by considering similar underlying mechanisms in a stochastic setup, which allows me to analyze the effect of countercyclical subsidies.

In the remaining literature close in spirit to my work, Annicchiarico et al. (2011), Annicchiarico & Rossi (2015) and Annicchiarico & Pelloni (2016) employ a New Keynesian model with endogenous growth as a result of learning-by-doing to study optimal monetary policy. Their work however has no scope for deliberate R&D investment nor

1946. The comparability of data may be affected by: a definitional change to an “establishment” basis of tabulation from a “reporting unit” concept prior to 1974; the determination of “active” status of an establishment prior to 1983; and changes in industrial classification beginning in 1988.”

firm dynamics. Benigno & Fornaro (2017) build a New Keynesian growth with innovating entrants as in Aghion & Howitt (1992) to study the effects of large policy interventions in persistent liquidity traps. Cozzi et al. (2017) estimate a business cycle model with Schumpeterian components such as creative destruction to assess the relative importance of several shocks on the recent performance of the US economy, but they abstract from welfare or policy analysis. Welfare considerations and optimal industrial policy are at the heart of paper by Atkeson & Burstein (2011), although their analysis employs a deterministic model. Finally, Nuño (2011) studies the effects of several static and countercyclical subsidy schemes in a stochastic Aghion & Howitt (1998) model and finds that a time-varying countercyclical subsidy to R&D has no positive effect on welfare. However, all of these papers abstract from endogenous R&D expenditures by incumbents over the business cycle, a crucial mechanism analyzed in this thesis.

Chapter 2

Business cycles, innovation and growth: welfare analysis

How (and if) should we foster innovations? Should we reduce entry barriers for new establishments, or maybe should we subsidize establishments investing in research and development? What are the consequences of those policies on economic growth rate and the effects of business cycles?

Economic literature has attempted to answer the above, and similar, questions. However, existing studies had to pick between analyzing those issues from the vantage points of either endogenous economic growth, or business cycles. This Chapter presents a novel, tractable approach that allows for analyzing the effects of aforementioned policies in a model featuring both economic growth and business cycle fluctuations. Thanks to that the results are more robust and it is possible to study the effects of additional policies that are absent in the extant literature.

This Chapter builds a model of heterogeneous, monopolistically competitive establishments (plants) that endogenously choose the intensity of research and development, while subject to endogenous entry and exit. The objective of this Chapter is to integrate several strands of literature on the long-lasting effects of temporary shocks and provide a new assessment of the welfare cost of business cycles, as well as novel guidance on industrial policy over the business cycle.

I find that business cycle fluctuations have a noticeable impact on the endogenous growth rates. Two main channels are responsible for this effect. First, incumbents behave procyclically, investing more in R&D in good times, and less in bad times. Second, net entry is also strongly procyclical. As the innovations performed by entrants tend to be more radical than those by incumbents, reduced entry rates put downward pressure on aggregate growth rates.

The results from the model indicate that almost 6% of a temporary shock is translated to the level shift in the Balanced Growth Path, and 4% of the shock is embedded within the first 5 years. This has significant welfare consequences, as the cost of business cycle fluctuations is of two orders of magnitude higher than in the exogenous growth variant of the model. The presence of large welfare effects and the ability to potentially affect the growth rates and volatility of the economy through appropriate industrial policy creates space for policy intervention via countercyclical subsidies. Of those the most positive welfare effect is achieved through countercyclical subsidies to incumbents' operating cost,

as it prevents excessive exits and encourages more R&D spending.

The Chapter is organized as follows. The next section describes the model, with particular emphasis on the problem of an incumbent establishment. The second section provides the solution for the Balanced Growth Path of the model economy to gain some intuition on the model mechanisms and discusses the stochastic solution procedure. The third section presents the data and calibration, as well as stochastic properties of the model economy in comparison to the data. This section offers also an application of the model to the aftermath of the Great Recession and points out to sources of seemingly permanent level shift in the US GDP. The fourth section is devoted to welfare analysis, providing an estimate of the welfare cost of business cycles for the US economy, an analysis of sensitivity of model outcomes to assumed parameter values, and a discussion on welfare improving subsidy schemes. The last section concludes.

2.1 Model

The model developed in this Chapter is mostly inspired by a closed economy version of the model sketched in the Endogenous Firm Productivity section of Melitz & Redding (2014), assuming very similar market structure and innovation process. Some inspiration is also drawn from Acemoglu et al. (2013), especially regarding the distinction between skilled and unskilled labor. This assumption effectively breaks the effect present in other endogenous growth models that since in recession the cost of labor is lower, it is optimal to increase R&D intensity.

The model focuses heavily on the cyclical behavior of the intermediate goods producers, and so I consciously keep the complexity of setup along other dimensions at a reasonable minimum. In the following exposition I employ the following notational convention. All aggregate and nominal variables are written in uppercase letters. Lowercase letters are reserved for real prices and variables related to individual establishments or households.

2.1.1 Households

There is a unit mass of households. As in Acemoglu et al. (2013), there are two types of workers: those that supply skilled and unskilled labor. Each household is modeled as a large family with a fixed mass s of skilled workers¹ and mass $1 - s$ of unskilled workers who pool their incomes and consume identical amounts regardless of their labor market status.

A representative household maximizes the following social welfare function which takes into account the utilities of both types of households' laborers with proportional weights:

$$U_0 = (1 - s) U_0^u + s U_0^s = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\theta}}{1-\theta} - (1-s) \psi_t^u \frac{(n_t^u)^{1+\kappa}}{1+\kappa} - s \psi_t^s \frac{(n_t^s)^{1+\kappa}}{1+\kappa} \right] \quad (2.1)$$

where U_0^u and U_0^s are the lifetime expected utilities of unskilled and skilled laborers, respectively, c_t denotes per capita consumption, while n_t^u and n_t^s denote labor supply

¹See Laincz & Peretto (2006) for empirical evidence that the share of employees engaged in R&D has been stationary in the time period between 1964-1999.

of unskilled and skilled laborers, respectively. Parameter β is the discount factor and the shape of the utility function is regulated by the inverse of elasticity of intertemporal substitution parameter θ and the inverse of Frisch elasticity parameter κ . Both ψ_t^u and ψ_t^s are normalization factors that grow together with the aggregate quality index of the economy Q_t and ensure that unskilled and skilled labor supply is equal to 1 along the Balanced Growth Path (BGP)².

I assume that physical capital plays no part in the production process³. The only asset in this economy in net positive supply are claims on the shares of establishments. Accordingly, the budget constraint of the household is constructed as follows:

$$c_t + p_t^{sh} sh_{t+1} = w_t^u n_t^u + w_t^s n_t^s + sh_t (p_t^{sh} + \Pi_t)$$

where p_t^{sh} is the real price of a share of firm portfolio that pays real dividends Π_t , sh_t is the mass of shares owned by the representative household, and w_t^u and w_t^s denote, respectively, real unskilled and skilled wages.

The first order conditions of the utility maximization problem reduce to the following two intratemporal equations and the Euler equation:

$$(1 - s) \psi_t^u (n_t^u)^\kappa = w_t^u c_t^{-\theta} \quad (2.2)$$

$$s \psi_t^s (n_t^s)^\kappa = w_t^s c_t^{-\theta} \quad (2.3)$$

$$1 = E_t \left[\beta \left(\frac{c_{t+1}}{c_t} \right)^{-\theta} \frac{p_{t+1}^{sh} + \Pi_{t+1}}{p_t^{sh}} \right]$$

The stochastic discounting kernel of the household is then defined as follows:

$$\Lambda_{t,t+i} = E_t \left[\left(\frac{c_{t+i}}{c_t} \right)^{-\theta} \right] \quad (2.4)$$

2.1.2 Final goods producer

The final goods producing sector is modeled as a single, representative, perfectly competitive firm that transforms a continuum of mass $M_t \in (0, 1)$ of intermediate good varieties⁴ into the final good using the CES aggregator:

$$Y_t = \left[\int_0^{M_t} y_t(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}$$

²While Boppart & Krusell (2016) show that balanced growth is possible when hours worked per worker are falling over time, a large body of research in macroeconomics employs utility functions of the convenient King et al. (1988) form which yield constant hours worked along the BGP. Trabandt & Uhlig (2011) prove that when the elasticity of intertemporal substitution differs from unity, only one functional form is characterized by both King-Plosser-Rebelo assumptions and constant Frisch elasticity of labor supply. Due to the restrictiveness of this specification, many authors, e.g. Mertens & Ravn (2011), employ a modeling shortcut in the form of time-dependent disutility of labor. Following this line of reasoning, I set the formula for labor disutility as $\psi_t^i = \psi^i Q_t^{1-\theta}$ (with $i = \{u, s\}$ and where Q is a trending variable discussed further), which ensures constant labor supply along the BGP.

³This assumption will be relaxed in the subsequent Chapters.

⁴The condition that the mass of intermediate goods varieties is bounded between 0 and 1 is supported by assuming that each individual possesses an idea for a differentiated product, but only a subset of those individuals are entrepreneurs and only a fraction of possible goods is actively produced.

where $y_t(i)$ denotes the quantity of i -th variety used in final good production process and $\sigma \in (1, \infty)$ is the elasticity of substitution between any two varieties. The standard solution of the cost minimization problem yields the price index of the final good as a function of the varieties' prices $P_t(i)$:

$$P_t = \left[\int_0^{M_t} P_t(i)^{1-\sigma} di \right]^{\frac{1}{1-\sigma}}$$

as well as the Hicksian demand function for the i -th variety:

$$y_t(i) = Y_t p_t(i)^{-\sigma} \quad (2.5)$$

where $p_t(i) = P_t(i) / P_t$ is the variety's price relative to the price index.

2.1.3 Intermediate goods producers

The intermediate goods producing sector is modeled as a single industry sector populated by monopolistically competitive continuum of mass M_t of active single-establishment firms⁵, each producing a distinct variety. Once an establishment hires f units of skilled labor (this can be thought of as managers and other non-production employees), it gains access to the following production function:

$$y_t(i) = Z_t q_t(i) n_t(i)$$

where Z_t is the stochastic aggregate productivity parameter, $q_t(i)$ is the quality level of i -th variety at time period t and $n_t(i)$ denotes the employment of unskilled labor.

It is straightforward to show that the optimal pricing strategy given flexible prices and the demand for an individual variety given by Equation 2.5 follows the standard constant mark-up pricing formula:

$$p_t(i) = \frac{\sigma}{\sigma - 1} \frac{w_t^u}{Z_t q_t(i)}$$

2.1.4 Aggregation

It is very convenient to introduce a measure of aggregate quality of intermediate goods produced in the economy. As in Melitz (2003), I assume that the distribution of establishment specific products' quality at time t is described by some probability density function $\mu_t(q)$ with support on a subset of $(0, \infty)$. Then one can define an aggregate quality index Q_t which is designed so that the aggregate state of the intermediate goods sector can be summarized as if it was populated by mass M_t of establishments all with quality level Q_t . The index is given by the following formula:

$$Q_t = \left[\int_0^\infty q^{\sigma-1} \mu_t(q) dq \right]^{\frac{1}{\sigma-1}} \quad (2.6)$$

⁵Klette & Kortum (2004) in a relatively similar setting show that the behavior of multi-product firms can be summarized as if they consisted of a number of independent product lines (establishments).

As a consequence, the aggregate output of the final good can be expressed as:

$$Y_t = Q_t Z_t M_t^{\frac{1}{\sigma-1}} N_t^u \quad (2.7)$$

where the dependence of output on M_t reflects the love-for-variety phenomenon and N_t^u is the aggregate unskilled labor supply.

The model features three aggregate state variables: aggregate quality level Q_t , stochastic productivity Z_t and active establishment mass M_t . The aggregate quality level evolution will be discussed at length in Subsection 2.1.7. Following the business cycles literature, I assume the exogenous common productivity shock to follow an AR(1) process in logs:

$$\ln Z_{t+1} = \rho_Z \ln Z_t + \varepsilon_{t+1} \quad (2.8)$$

where ρ_Z is the autoregressivity of the process and ε_{t+1} is a random, normally distributed innovation with mean 0 and standard deviation σ_Z .

The mass of active establishments evolves according to the following, endogenously determined law of motion:

$$M_{t+1} = M_t - M_t^x - \delta_t (M_t - M_t^x) + M_t^e \quad (2.9)$$

where M_t^x denotes mass of establishments exiting “voluntarily” due to product obsolescence, δ_t is the state-dependent, endogenous probability of receiving an exit shock and M_t^e is the mass of successful entrants who attempted entry at time period t . Due to the presence of fixed costs, the mass of active establishments stabilizes around the value characteristic for the BGP, although it fluctuates over the business cycle. The only source of sustained long-run economic growth is the continuing improvement in the aggregate quality level.

2.1.5 Incumbents

2.1.5.1 Research and development

Each incumbent establishment can engage in R&D activities in order to attempt to raise the quality of its variety in the next period. Innovations performed by incumbents should be interpreted as incremental, rather than radical. The success probability function of an establishment is modeled as in Pakes & McGuire (1994) and Ericson & Pakes (1995):

$$\chi_t(i) = \frac{ax_t(i)}{1 + ax_t(i)}$$

where $\chi_t(i)$ denotes the probability of making a quality improvement, and a is a parameter that describes the efficiency of R&D input $x_t(i)$ in generating improvements. The R&D input has the following formula:

$$x_t(i) = \frac{n_t^x(i)}{(q_t(i)/Q_t)^{\sigma-1}}$$

where $n_t^x(i)$ denotes skilled R&D labor input and $(q_t(i)/Q_t)^{\sigma-1}$ is a relative quality adjustment factor. As a consequence, establishments with higher quality products need

to employ more R&D labor to have the same success probability as their lower quality competitors.

The logic behind introducing this adjustment is based on two reasons. First, since the incumbents' innovations are incremental, a lot of those improvements stem from imitation rather than pure innovation. It is then reasonable that establishments producing low quality varieties have a bonus due to their distance from the “average” quality frontier, since they can easily imitate what the establishments producing higher quality varieties do. Conversely, establishments far ahead with respect to their products' quality have little opportunities to imitate, and rather have to innovate themselves, raising the input requirement.

The second reason is empirical, and relates to the Gibrat's law (Gibrat (1931)), which postulates that there is no correlation between firm size (which in the model is directly related to quality) and firm growth rates (which in the model result from the innovation success probability χ). Without the adjustment, establishments producing higher quality varieties would have comparative advantage of performing R&D relative to ones producing varieties of lower quality. However, empirical evidence on the evolution of firms shows that either the Gibrat's law cannot be rejected for large enough firms (see e.g. Hall (1987)) or that the larger firms have slower rates of growth (see e.g. Evans (1987), Dunne et al. (1989) or Rossi-Hansberg & Wright (2007)).

It is convenient to introduce a new variable $\phi_t(i)$ for this relative quality adjustment factor, so that:

$$\phi_t(i) \equiv (q_t(i)/Q_t)^{\sigma-1}$$

Given the above functions, one can derive the demand for R&D labor as a function of target success probability $\chi_t(i)$:

$$n_t^x(i) = \frac{1}{a} \frac{\chi_t(i)}{1 - \chi_t(i)} \phi_t(i)$$

and the real cost of innovation equals the R&D employment times the real skilled wage.

Making further use of the relative quality variable $\phi_t(i)$, the real operating profit of an establishment at time t can be rewritten as:

$$\pi_t^o(i) = \frac{Y_t}{\sigma M_t} \phi_t(i) - w_t^s f$$

and the real profit function, after taking into account R&D expenditures, is given by:

$$\pi_t(i) = Y_t \left[\left(\frac{1}{\sigma M_t} - \frac{\omega_t}{a} \frac{\chi_t(i)}{1 - \chi_t(i)} \right) \phi_t(i) - \omega_t f \right] \quad (2.10)$$

where $\omega_t \equiv w_t^s/Y_t$ denotes the ratio between the skilled wage and aggregate output.

2.1.5.2 Recursive formulation

I will now recast the incumbents' problem into dynamic programming form. Since all intermediate goods producers that share the same value of ϕ will make the same decisions, I drop the subscript i . Moreover, if the expected continuation value is negative, an establishment will exit at the end of the current period. An incumbent establishment

maximizes its real discounted stream of profits, conditional on the expected paths of state variables and the control variable χ :

$$V_t(\phi_t) = \max_{\chi_t \in [0,1]} \{ \pi_t(\phi_t, \chi_t) + \max \{0, E_t [\beta \Lambda_{t,t+1} (1 - \delta_t) V_{t+1}(\phi_{t+1} | \phi_t, \chi_t)] \} \}$$

where $\Lambda_{t,t+1}$ is the stochastic discount factor consistent with households' valuation of current and future marginal utility from consumption (Equation 2.4) and the relative quality of a variety in $t + 1$ period is subject to the following lottery⁶:

$$\phi_{t+1} = \begin{cases} \iota \phi_t / \eta_t & \text{with probability } \chi_t \\ \phi_t / \eta_t & \text{with probability } 1 - \chi_t \end{cases}$$

where ι denotes the size of the innovative step and η is the rate of growth of the aggregate quality index (raised to the $\sigma - 1$ power):

$$\eta_t \equiv \left(\frac{Q_{t+1}}{Q_t} \right)^{\sigma-1}$$

Here I assume that the value of η is taken as given by the individual establishments. This assumption is justified in the case of a continuum of atomistic agents⁷.

As is common in the dynamic programming literature, I shall proceed with the following notational convention. Variables without the time subscript shall denote time period t variables, while variables with a prime ' shall denote the $t + 1$ variables. Let the vector $\mathbf{S} \equiv \{Q, Z, M\}$ denote the aggregate state of the economy.

Due to the presence of trending variables in the original formulation of the problem, I shall also normalize the value function by expressing it as a product of aggregate output, dependent only on the economy's state, and time-independent component v , which depends both on the aggregate state and idiosyncratic relative quality:

$$Y(\mathbf{S}) v(\phi, \mathbf{S}) = \max_{\chi \in [0,1]} \left\{ Y(\mathbf{S}) \left[\left(\frac{1}{\sigma M} - \frac{\omega(\mathbf{S})}{a} \frac{\chi}{1-\chi} \right) \phi - \omega(\mathbf{S}) f \right] + \max \{0, E [\beta \Lambda(\mathbf{S}, \mathbf{S}') (1 - \delta(\mathbf{S})) Y(\mathbf{S}') v(\phi', \mathbf{S}')] \} \right\}$$

Both sides of the above expression can be divided by the aggregate output, reducing the problem to the form that can be handled via standard contraction mapping procedures. Note that the “relative” skilled wage ω is established on a period by period basis via the skilled labor market equilibrium, upon which I expand later. Moreover, the state space is effectively reduced to contain only two aggregate variables: stochastic productivity Z and active establishment mass M . The normalized value function of an establishment thus equals:

$$v(\phi, \mathbf{S}) = \max_{\chi \in [0,1]} \left\{ \left[\left(\frac{1}{\sigma M} - \frac{\omega(\mathbf{S})}{a} \frac{\chi}{1-\chi} \right) \phi - \omega(\mathbf{S}) f \right] + \max \{0, E [\beta \Lambda(\mathbf{S}, \mathbf{S}') \gamma(\mathbf{S}, \mathbf{S}') (1 - \delta(\mathbf{S})) v(\phi', \mathbf{S}')] \} \right\} \quad (2.11)$$

⁶The underlying quality levels evolve according to the lottery:

$$q_{t+1} = \begin{cases} \iota^{1/(\sigma-1)} q_t & \text{with probability } \chi_t \\ q_t & \text{with probability } 1 - \chi_t \end{cases}$$

⁷For the case of a finite number of firms, see the “oblivious equilibrium” concept developed in Weintraub et al. (2008).

where $\gamma(\mathbf{S}, \mathbf{S}') \equiv Y(\mathbf{S}')/Y(\mathbf{S})$ denotes the growth rate of aggregate output between two subsequent periods.

2.1.5.3 Tractability and obsolescence

For large enough ϕ the probability that an establishment will in the future exit due to obsolescence is negligible. The problem then simplifies to:

$$v(\phi, \mathbf{S}) = \max_{\chi \in [0,1]} \left\{ \left[\left(\frac{1}{\sigma M} - \frac{\omega(\mathbf{S})}{a} \frac{\chi}{1-\chi} \right) \phi - \omega(\mathbf{S}) f \right] + E[\beta \Lambda(\mathbf{S}, \mathbf{S}') \gamma(\mathbf{S}, \mathbf{S}') (1 - \delta(\mathbf{S})) v(\phi', \mathbf{S}')] \right\} \quad (2.12)$$

and in optimum the R&D success probability does not depend on the quality of establishments' variety. The above problem is affine in ϕ , as both the immediate payoff (profit) function and the expected continuation value function are affine in ϕ ⁸.

On the low end of the ϕ distribution, if an establishment chooses to exit, it does not invest in R&D at all ($\chi = 0$) and its current value is also affine in ϕ :

$$v(\phi, \mathbf{S}) = \frac{1}{\sigma M} \phi - \omega(\mathbf{S}) f \quad (2.13)$$

The true value function is nonlinear in the area where establishments are not yet willing to exit (the expected continuation value is still positive), but the probability that they will choose to do so in the future is not negligible. This leads them to invest in R&D less than their higher quality competitors do. Furthermore, some establishments may choose not to invest in R&D at all if their expected continuation value is positive but small. Keeping this non-linearity would make the entire distribution of establishments' qualities (or at least a set of its moments) a state variable, and would require much more advanced numerical techniques to solve⁹.

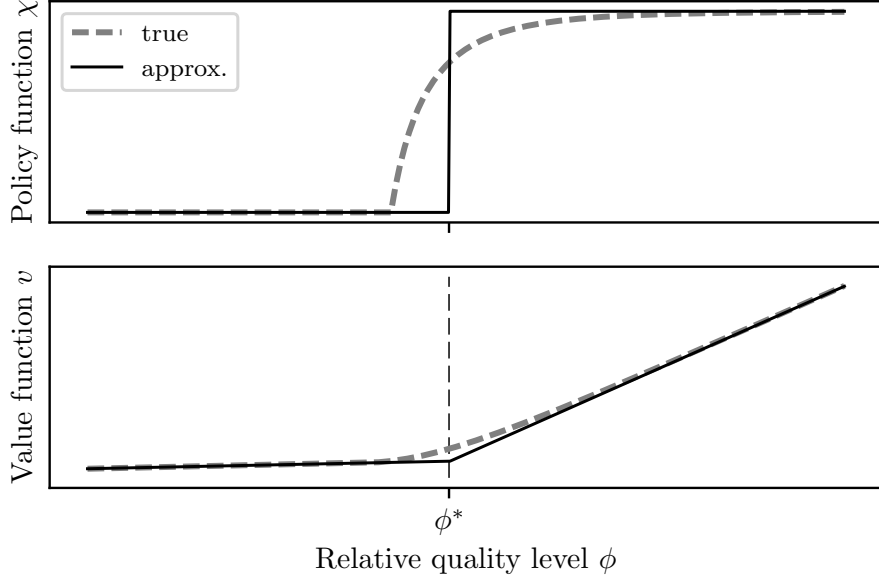
In order to improve the problem's tractability and sidestep the issue of distribution tracking, I consider an approximation of the true problem, as depicted in Figure 2.1. I extend the linear parts of the true value function and construct an approximating piecewise linear value function. This simplification implies that the approximate policy function becomes a piecewise constant function of ϕ , with all continuators choosing exactly the same R&D success probability χ . The establishments that are active but will exit in the current period choose not to innovate at all. The Appendix contains a proof¹⁰ that if the innovation probabilities are independent of establishment size but there is a lower bound on establishment quality, the distribution of establishment qualities converges in the upper tail to an ergodic Pareto distribution regardless of the initial distribution of entrants' qualities.

⁸It is straightforward to prove that the sum of two affine functions is also an affine function. Consider $f(x) = a + bx$ and $g(x) = c + dx$. Then their sum, $h(x) = (a + c) + (b + d)x$, is clearly an affine function as well. This procedure generalizes to the case of an infinite sum.

⁹Krusell & Smith (1998) is the seminal paper in the literature on heterogeneous agents subject to both idiosyncratic and aggregate shocks.

¹⁰The proof is based on the Web Appendix for the Melitz & Redding (2014).

Figure 2.1: Accurate and approximated value and policy functions



Let ϕ^* denote a level of relative quality such that an establishment is indifferent between exiting at the end of the current period and continuing into the next period, conditional on setting the common R&D success probability χ . This cutoff value is given implicitly by the following condition:

$$\frac{\omega(\mathbf{S})}{a} \frac{\chi}{1-\chi} \phi^* = \mathbb{E} \left[\beta \Lambda(\mathbf{S}, \mathbf{S}') \gamma(\mathbf{S}, \mathbf{S}') (1 - \delta(\mathbf{S})) v((\phi^*)', \mathbf{S}') \right] \quad (2.14)$$

The cyclical movements in ϕ^* are responsible for the endogenous “voluntary” exit margin. To provide a closed form equation of this process, I will assume that the distribution of establishment qualities follows exactly the Pareto distribution with power parameter equal to 1 for the entirety of its support¹¹. The mass of establishments exiting due to obsolescence is equal to:

$$M_t^x = M_t (1 - \chi_t) \left(1 - \frac{\phi_{t-1}^*}{\phi_t^* \eta_{t-1}} \right) \quad (2.15)$$

2.1.6 Entrants

There is an unconstrained mass of prospective entrants. Within each period, they can exert innovative effort to attempt to successfully enter the market. To do that, they hire skilled labor in the similar manner the incumbents do. I also assume that prospective

¹¹This assumption is common in the firm size distribution literature. For empirical support see e.g. Axtell (2001). Note that this is the only place where I need to assume a specific functional form for the distribution of quality levels. All other results hold for generic distributions, although only the distributions whose upper tail converge to Pareto are consistent with the other assumptions of the model.

entrants face positive skilled labor fixed cost, f^e . The real cost of attempting entry conditional on success probability χ_t^e is thus equal to:

$$tc_t^e = w_t^s \left(f^e + \frac{1}{a^e} \frac{\chi_t^e}{1 - \chi_t^e} \right)$$

Upon successful entry an entrant draws the initial quality level from the distribution of incumbent qualities, scaled up by $[\sigma/(\sigma-1)]^{1/(\sigma-1)}$ ¹². This assumption reflects the fact that innovations developed by entrants tend to be more radical, as stressed by i.a. Acemoglu & Cao (2015) and Garcia-Macia et al. (2016). A successful entrant begins operation at the beginning of the next period, and discounts this fact accordingly. The expected value of entry is expressed as:

$$V_t^e = \max_{\chi_t^e \in [0,1]} \left\{ -Y_t \omega_t \left(f^e + \frac{1}{a^e} \frac{\chi_t^e}{1 - \chi_t^e} \right) + \chi_t^e E_t [\beta \Lambda_{t,t+1} V_{t+1}(\phi_{t+1}^e)] \right\}$$

where ϕ_{t+1}^e is the relative quality draw upon entry. Again, using the dynamic programming notation and after normalization with aggregate output, one obtains:

$$v^e(\mathbf{S}) = \max_{\chi^e \in [0,1]} \left\{ \begin{array}{c} -\omega(\mathbf{S}) \left(f^e + \frac{1}{a^e} \frac{\chi^e}{1 - \chi^e} \right) \\ + \chi^e E [\beta \Lambda(\mathbf{S}, \mathbf{S}') \gamma(\mathbf{S}, \mathbf{S}') v((\phi^e)', \mathbf{S}')] \end{array} \right\} \quad (2.16)$$

Since the pool of entrants is unbounded, in each state of the economy the following free entry condition applies:

$$v^e(\mathbf{S}) \leq 0 \quad (2.17)$$

where in the case of a negative value of entry the mass of entrants is 0. Finally, if the mass of successful entrants is denoted by M^e and the success probability is χ^e then in equilibrium the mass of prospective entrants is determined by M^e/χ^e in case of positive χ^e and 0 otherwise.

2.1.6.1 Creative destruction

Although entry is undirected, there is a possibility of an entrant leapfrogging over an incumbent. To model that possibility, I assume that the space of all possible varieties occupies a unit interval, and active establishments occupy its subset M_t . This can be justified by assuming that each individual person has a potential business idea, but only a subset of them is realized at any given time. It is then natural that the mass of potential ideas is proportional to (here equal to) the unit household mass. A successful entrant draws its location from the entire interval, and a fraction M_t of all entrants replaces previously active establishments¹³.

To account for the fact that the entrants who replace the incumbents have leapfrogged over them, I assume that the quality advantage of entrants is high enough to ensure no

¹²This assumption ensures that each variety is produced by a single establishment and that no establishment needs to resort to limit pricing as opposed to the regular markup pricing.

¹³One could make an alternative assumption that the mass of potential varieties is unbounded. Although it is a frequent assumption in case of expanding varieties models, it would be hard to reconcile with creative destruction.

limit pricing. Thus, an establishment in which an incumbent successfully innovated but was replaced by an entrant will be characterized by $[\iota \cdot \sigma / (\sigma - 1)]^{1/(\sigma-1)}$ times higher quality compared to the previous period and the establishment where an incumbent was unsuccessful in innovating and got replaced will be characterized by $[\sigma / (\sigma - 1)]^{1/(\sigma-1)}$ times higher quality. Therefore, the expected relative quality of an entrant is equal to:

$$\phi^e = \frac{\sigma}{\sigma - 1} \quad (2.18)$$

It is now possible to characterize the aggregate establishment mass dynamics. Incumbents exit for three reasons, which I assume to happen in the following order. First, their relative quality becomes low enough that they decide to exit due to the obsolescence of their product. Second, they may receive an exogenous exit shock, δ^{exo} . Finally, their establishment may be creatively destroyed by a successful entrant.

Successful entrants' varieties may or may not overlap with already produced varieties. Creative destruction is a churning process, where outflows and inflows are by definition equal. Thus, I only need to account for exogenous exit and entry into previously inactive varieties:

$$M_{t+1} = (1 - \delta^{exo}) (M_t - M_t^x) + [1 - (1 - \delta^{exo}) (M_t - M_t^x)] M_t^e \quad (2.19)$$

where I use the fact that the mass of active establishments just before entry is equal to $(1 - \delta^{exo}) (M_t - M_t^x)$.

A comparison between Equations 2.9 and 2.19 reveals that the following relationship holds:

$$\delta_t = 1 - (1 - \delta^{exo}) (1 - M_t^e) \quad (2.20)$$

2.1.7 Aggregate quality evolution

Following Melitz (2003), I consider the current period distribution of quality levels $\mu_t(q)$ to be a truncated part of an underlying distribution $g_t(q)$, so that:

$$\mu_t(q) = \begin{cases} 1/[1 - G_t(q_{t-1}^*)] g_t(q) & \text{if } q \geq q_{t-1}^* \\ 0 & \text{otherwise} \end{cases}$$

where the lowest quality level among all active establishments at period t is equal to previous period's cutoff quality level q_{t-1}^* ¹⁴. The aggregate quality level defined by Equation 2.6 can be rewritten as:

$$Q_t = \left[\frac{1}{1 - G_t(q_{t-1}^*)} \int_{q_{t-1}^*}^{\infty} q^{\sigma-1} g_t(q) dq \right]^{\frac{1}{\sigma-1}}$$

There are two mechanisms that affect the distribution of incumbents' quality levels. First, lowest quality establishments exit, raising the next period's lowest quality level to q_t^* . Second, while $1 - \alpha_t$ share of incumbents do not change their quality levels, share χ_t of incumbents innovate successfully and raise their quality levels by $\iota^{1/(\sigma-1)}$. The aggregate

¹⁴The absolute level of cutoff quality can be recovered from the relative cutoff quality via the formula $q_t^* = (\phi_t^*)^{1/(\sigma-1)} Q_t$.

quality level after exits¹⁵ and innovation resolution but before entry can be expressed as follows:

$$Q_t^* = \left[(1 - \chi_t + \chi_t \iota) \frac{1}{1 - G_t(q_t^*)} \int_{q_t^*}^{\infty} q^{\sigma-1} g_t(q) dq \right]^{\frac{1}{\sigma-1}}$$

Entrants draw their quality levels from the above distribution, upscaled by factor $\sigma/(\sigma - 1)$. The share of new establishments in the $t + 1$ period equals M_t^e/M_{t+1} and the aggregate quality level in $t + 1$ is described by:

$$Q_{t+1} = \left[(1 - \chi_t + \chi_t \iota) \left(1 - \frac{M_t^e}{M_{t+1}} + \frac{M_t^e}{M_{t+1}} \frac{\sigma}{\sigma - 1} \right) \frac{1}{1 - G_t(q_t^*)} \int_{q_t^*}^{\infty} q^{\sigma-1} g_t(q) dq \right]^{\frac{1}{\sigma-1}}$$

Finally one can derive the transformed aggregate growth rate η_t ¹⁶:

$$\eta_t = \left(\frac{Q_{t+1}}{Q_t} \right)^{\sigma-1} = (1 - \chi_t + \chi_t \iota) \left(1 - \frac{M_t^e}{M_{t+1}} + \frac{M_t^e}{M_{t+1}} \frac{\sigma}{\sigma - 1} \right) \quad (2.21)$$

Ceteris paribus, aggregate quality grows faster when incumbents engage more intensively in R&D and when entry rates are elevated. In general equilibrium those two forces moderate each other, as both incumbents and potential entrants compete for scarce skilled labor and incumbents face a direct threat of being creatively destroyed by entrants, reducing their willingness to innovate.

2.1.8 Market clearing

There are two labor markets that clear each period. Since the aggregate mass of unskilled workers is equal to $1 - s$, the following relationship between aggregate and individual labor supply holds:

$$N_t^u = (1 - s) n_t^u \quad (2.22)$$

As the production function depends linearly on unskilled labor, the unskilled wage is independent of the supply side. Unskilled labor supply is determined via the unskilled intratemporal condition (Equation 2.2) and is given by:

$$n_t^u = \left[\frac{\sigma - 1}{\sigma} Z_t^{1-\theta} M_t^{\frac{1-\theta}{\sigma-1}} (1 - s)^{-(1+\theta)} / \psi^u \right]^{\frac{1}{\kappa+\theta}} \quad (2.23)$$

where I use the assumption about the labor disutility normalization factors that $\psi_t^u = \psi^u Q_t^{1-\theta}$ and with ψ^u chosen so that along the BGP individual unskilled labor supply is equal to unity.

As opposed to the unskilled labor market, which behaves rather mechanically, finding equilibrium in the skilled labor market is more involved. First, using the skilled intratemporal condition (Equation 2.3) I obtain the relationship between skilled labor supply and

¹⁵Note also that due to the assumption that probabilities of both exogenous exit shock and creative destruction do not depend on establishment quality, they do not affect the quality distribution.

¹⁶If the density g is assumed to be Pareto, which is immutable with respect to the truncation point, the argument holds with equality. Otherwise, this result is an approximation relying on assuming that the shifts in cutoff quality levels are small enough to not affect significantly the entire distribution.

the relative skilled wage ω_t :

$$n_t^s = \left[\omega_t \left(Z_t M_t^{\frac{1}{\sigma-1}} N_t^u \right)^{1-\theta} / (s\psi^s) \right]^{\frac{1}{\kappa}} \quad (2.24)$$

where again I use the assumption about the labor disutility normalization factors that $\psi_t^s = \psi^s Q_t^{1-\theta}$ and with ψ^s chosen so that along the BGP individual skilled labor supply is equal to unity.

Second, the relative skilled wage influences the R&D intensity choices made by incumbents and prospective entrants. The skilled labor demand, which implicitly depends on ω_t , is expressed as:

$$N_t^s = M_t f + (M_t - M_t^x) \left(\frac{1}{a} \frac{\chi_t}{1 - \chi_t} \right) + \frac{M_t^e}{\chi_t^e} \left(\frac{1}{a^e} \frac{\chi_t^e}{1 - \chi_t^e} \right) \quad (2.25)$$

and emerges from three sources: “managerial” demand from all active establishments, R&D demand from active and non-obsolete establishments, and “managerial” and R&D demand from prospective entrants. In equilibrium, relative skilled wage ω_t adjusts to equate skilled labor demand and supply, so that:

$$N_t^s = s n_t^s \quad (2.26)$$

Finally, the goods market clears:

$$Y_t = (1 - s) c_t + s \cdot c_t \quad (2.27)$$

2.2 Solution procedure

2.2.1 Balanced growth path

To provide more intuition on the working of the model and discuss its stability properties, I first present the salient parts of the solution for the BGP which can be to a large extent derived analytically. The labor market is assumed to equilibrate when both individual skilled and unskilled labor equals to 1 and the households’ stochastic discount factor (Equation 2.4) can be expressed as:

$$\Lambda = \gamma^{-\theta}$$

where γ is the gross rate of growth of output along the BGP.

2.2.1.1 Incumbents

I exploit the property that the value function of the incumbent with high enough ϕ , given by Equation 2.12, is affine. As along the BGP the reduced aggregate state is time-invariant, all variables present in the incumbents’ problem are constant, and the value function can be stated as:

$$A + B\phi = \max_{\chi \in [0,1]} \left\{ \begin{aligned} & \left[\left(\frac{1}{\sigma M} - \frac{\omega}{a} \frac{\chi}{1-\chi} \right) \phi - \omega f \right] \\ & + \xi \left[\chi \left(A + B \frac{\phi}{\eta} \right) + (1 - \chi) \left(A + B \frac{\phi}{\eta} \right) \right] \end{aligned} \right\}$$

where $\xi \equiv \beta\gamma^{1-\theta} (1 - \delta)$ is the effective incumbents' discount factor.

The resulting first order and envelope conditions are:

$$0 = -\frac{\omega}{a} \frac{1}{(1-\chi)^2} \phi + \xi B \frac{\phi(\iota-1)}{\eta}$$

$$B = \left(\frac{1}{\sigma M} - \frac{\omega}{a} \frac{\chi}{1-\chi} \right) + \xi B \frac{\chi(\iota-1) + 1}{\eta}$$

There are two approaches that can be used to solve this problem – partial equilibrium and explicit solutions¹⁷. Both solutions are derived in the Appendix. Here I present only the result of the partial equilibrium solution where an establishment treats “aggregate” and individual R&D success probabilities as separate objects. This allows to focus on the intuition behind the factors driving incumbents' decisions. The optimal R&D success probability equals:

$$\chi = \frac{\frac{1}{\sigma M} \frac{a}{\omega} - \frac{1-\xi\zeta}{\xi} \frac{\eta}{\iota-\zeta\eta}}{1 + \frac{1}{\sigma M} \frac{a}{\omega}}$$

where $\zeta \equiv [\chi(\iota-1) + 1]/\eta$ can be interpreted loosely as the contribution of incumbents to raising the aggregate quality level and is treated as given by an individual establishment.

Note that while χ is smaller than one for any economically sensible parametrization, one needs to ensure that it is positive. For that to be true, the sign of the numerator has to be positive, which implies that the real revenue for the average establishment needs to be high enough to justify investment in R&D.

The following set of partial derivatives conforms with intuition and previous research in endogenous growth theory literature. Incumbents invest less in R&D when their monopolistic power is low, market is fragmented among many firms and when the R&D labor cost is high. Incumbents invest more in R&D when R&D labor is more productive, discount factor is closer to unity and the innovative step size is high.

$$\begin{array}{lll} \frac{\partial \chi}{\partial \sigma} < 0, & \frac{\partial \chi}{\partial M} < 0, & \frac{\partial \chi}{\partial \omega} < 0 \\ \frac{\partial \chi}{\partial a} > 0, & \frac{\partial \chi}{\partial \xi} > 0, & \frac{\partial \chi}{\partial \iota} > 0 \end{array}$$

A second set of partial derivatives relates to the stability properties of the problem. Partial derivatives with respect to transformed aggregate quality growth rate and incumbents' contribution to growth are negative¹⁸ which ensures that the BGP is a stable equilibrium of the system and can be reached by a trial-and-error process (i.e. the equilibrium is learnable¹⁹):

¹⁷Obviously the problem can be also solved numerically via value function iteration which can then be subsequently extended to the stochastic environment case.

¹⁸The partial derivative w.r.t. incumbents' contribution to growth is negative for reasonable parametrizations. The formal condition for the negativity is $\xi\iota > \eta$, which is easily satisfied when the effective discount factor is close to unity.

¹⁹For a game-theoretic discussion on learning in non-cooperative games see e.g. Milgrom & Roberts (1990).

$$\frac{\partial \chi}{\partial \eta} < 0, \quad \frac{\partial \chi}{\partial \zeta} < 0$$

Once the optimal R&D success probability level is found, the parameters of the closed form of the value function can be easily recovered:

$$B = \frac{\frac{1}{\sigma M} - \frac{\omega}{a} \frac{\chi}{1 - \chi}}{1 - \xi \zeta}, \quad A = \frac{\omega f}{1 - \xi}$$

2.2.1.2 Entrants

The problem of entrants from Equation 2.16 remains unchanged, except for the fact that growth rates and entry value are known with certainty:

$$v^e = \max_{\chi^e \in [0,1]} \left\{ -\omega \left(f^e + \frac{1}{a^e} \frac{\chi^e}{1 - \chi^e} \right) + \chi^e \xi^e (A + B \phi^e) \right\}$$

where $\xi^e \equiv \beta \gamma^{1-\theta}$ is the effective entrants' discount factor.

The first order condition for potential entrants results in:

$$\chi^e = 1 - \sqrt{\frac{\omega}{a^e \xi^e v(\phi^e)}}$$

that is, entry success probability is higher when R&D labor is more productive, discount factor is closer to unity and the expected entry value is higher. Entry success probability is lower when R&D labor is more costly.

The free entry condition, which in BGP has to hold with equality, pins down the entry success probability in equilibrium:

$$\chi^e = \frac{a^e f^e \pm \sqrt{a^e f^e}}{(a^e f^e - 1)}$$

In the above expression I need to assume that $a^e f^e > 1$ and then only one root lies within the unit interval. Note that entry success probability along the BGP depends neither on the expected entry value nor on the R&D labor cost. This result does not carry over outside the BGP and in general changes in entry rates are driven both by the extensive (mass of potential entrants) and intensive (success probability) margins.

2.2.1.3 General equilibrium

To close the model, I need to pin down the variables responsible for establishment dynamics, which are determined by the following system of equations:

$$\begin{aligned} M^x &= (1 - \chi) (1 - 1/\eta) M \\ M^e &= \delta M + (1 - \delta) M^x \\ \delta &= 1 - (1 - \delta^{exo}) (1 - M^e) \\ \eta &= (1 + \chi (\iota - 1)) (1 + \delta / (\sigma - 1)) \end{aligned}$$

Finally, the skilled labor market clears. Since by assumption the individual skilled labor supply along the BGP is unity, the aggregate skilled labor supply equals the mass of skilled households s and the equilibrium in the skilled labor markets occurs whenever:

$$s = Mf + (M - M^x) \left(\frac{1}{a} \frac{\chi}{1 - \chi} \right) + \frac{M^e}{a^e} \left(f^e + \frac{1}{a^e} \frac{\chi^e}{1 - \chi^e} \right)$$

Thanks to the favorable stability properties of the model, the solution can be easily found. Using an initial guess for “relative” skilled wage rate ω , endogenous exit probability δ and the mass of active establishments M , one can iterate the system forward until convergence is reached. I keep iterating as long as the L^∞ norm between subsequent iterations is higher than 10^{-12} .

2.2.2 Global solution

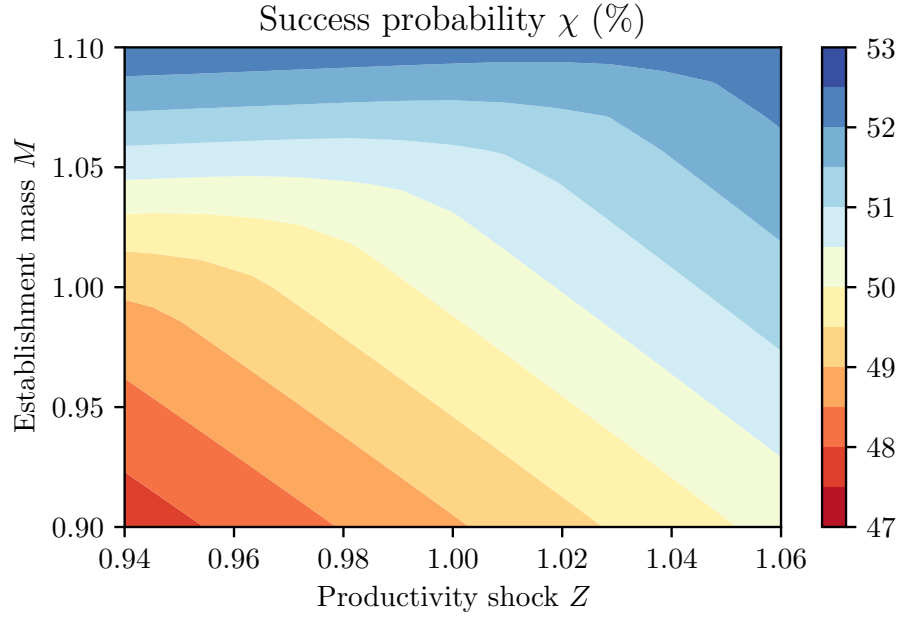
Under stochastic environment obtaining analytical results is not possible. Therefore, I solve the model using global methods, and employ stochastic value function iteration. The two state variables, exogenous aggregate productivity level Z and endogenous mass of active establishments M are discretized on a 15 by 15 grid. The grid for establishments spans the range of $\pm 10\%$ deviation from the BGP value.

The stochastic process for the changes in aggregate productivity is recast in the form of a Markov chain. Following the analysis in Kopecky & Suen (2010), I employ the method proposed by Rouwenhorst (1995), instead of more popular methods by Tauchen (1986) or Tauchen & Hussey (1991), as the former generates a better approximation to the underlying continuous process for highly persistent ($\rho_Z > 0.9$) processes. For the chosen parameters of the productivity process ($\rho_Z = 0.95$, $\sigma_Z = 0.0055$) and the desired grid density the Rouwenhorst (1995) method generates a grid for aggregate productivity level spanning the range of (0.93, 1.07).

The initial values for the endogenous variables are set to be equal to their BGP values. The stochastic value functions given by Equations 2.12 and 2.16 are iterated over each point on the grid as long as the L^∞ norm between subsequent iterations is higher than 10^{-9} . Also, for each point on the grid the general equilibrium consistency is ensured.

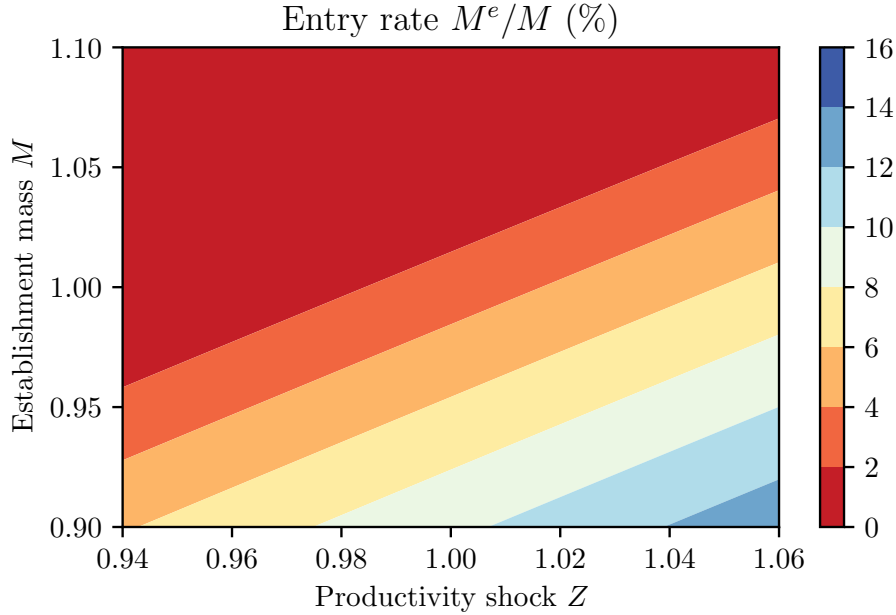
The resulting policy functions, conditional on assumed parameter values discussed at length in Subsection 2.3.1, are displayed in subsequent figures. Figure 2.1 demonstrates the dependence of the incumbents’ R&D success probability χ on the state variables, Z and M . The success probability increases with the value of the temporary productivity shock and decreases with the mass of active establishments. The intuition for the former is straightforward – when current stochastic productivity is higher than average, it is expected to be higher than average also in the future. The latter result seems counter-intuitive at first. However, whenever the mass of active establishments is lower than average, entry is more attractive. Therefore, incumbents face higher risk of being creatively destroyed, which lowers their effective discount factor, decreasing incentives to innovate.

Figure 2.1: Incumbents' policy function χ (%)



The policy function of potential entrants, expressed in terms of desired entry rate, is depicted in Figure 2.2. As could be expected, entry is more desirable when current productivity is higher than average and the mass of active establishments is lower than average. Note that the combination of low productivity and high mass of active establishments may make entry unattractive enough so that no potential entrant is willing to invest in R&D (upper left portion of the graph). This creates a non-linearity in the policy functions of both entrants and incumbents, and emphasizes the usefulness of the global solution of the model.

Figure 2.2: Entrants' policy function (expressed as desired entry rate M^e/M , %)



2.3 Data and results

2.3.1 Data and calibration

The data used in this Chapter come from four major sources. The primary source of data on establishment dynamics comes from the US Bureau of Labor Statistics (BLS) Business Employment Dynamics (BDM) database. The BDM, based on the Quarterly Census on Employment and Wages (QCEW), records changes in the employment level of more than 98% of economic entities in the US. Unfortunately, the data series is relatively short, starting as late as of 1992q3. Supplementary sources include the data from US Bureau of Economic Analyses (BEA), National Science Foundation (NSF) and County Business Patterns (CBP).

In the BDM, an establishment is defined as an economic unit that produces goods or services, usually at a single physical location, and engages in one, or predominantly one, activity for which a single industrial classification may be applied²⁰. Thus an establishment, as measured by the BLS, corresponds quite closely to the theoretical concept of establishment considered in the model.

Expansions (contractions) are defined as units with positive employment in the third month in both the previous and current quarters, with a net increase (decrease) in employment over this period. Viewed through the lens of the model, expansions are the result of a successful innovation, while contractions are a consequence of being unable to innovate and thus declining relative quality level. Openings are defined as establishments

²⁰This and the following definitions are quoted from the Business Employment Dynamics Technical Note, available at <http://www.bls.gov/news.release/cewbd.tn.htm>.

with either positive third month employment for the first time in the current quarter, with no links to the prior quarter, or with positive third month employment in the current quarter following zero employment in the previous quarter. Closings are defined as establishments either with positive third month employment in the previous quarter, with no employment or zero employment reported in the current quarter.

The problem with using these statistics directly is that both openings and closings are an upward biased measure of “true” entry and exit patterns, as they are very sensitive to seasonal employment patterns. To correct for this issue, BLS produces data on establishment births and deaths, which are a subset of openings and closings, controlled for re-openings and temporary shutdowns via “waiting” for three quarters for status confirmation. While this correction introduces some discrepancies in the aggregate data, the gains from using data series that are closer to the model objects should significantly outweigh the associated cost.

The model is calibrated to replicate key features of the US economy. The model BGP outcomes are compared to the long-run averages of the corresponding objects in the US data. Several parameters, such as the discount factor or the intertemporal elasticity of substitution (IES) are taken from literature.

The parameter value for the inverse of Frisch utility is non-standard, and the particular choice is dictated by the need to obtain sufficiently strong, positive reaction of labor supply in face of the productivity shock. Nevertheless, the resulting volatility of hours is still much smaller than that of output, a ubiquitous issue in the business cycle literature. The standard deviation of the productivity shock is smaller than usually adopted in the literature, and this is due to the model having stronger amplification properties compared to the baseline real business cycle models.

The parameter pinning down the share of skilled workers in the economy is chosen to be well inside the plausible range of values discussed by Acemoglu et al. (2013)²¹ and close to their estimated value of 7.8%. Since in their model a part of innovations arise spontaneously (due to e.g. learning by doing) and in this model all innovations result from deliberate R&D investment, it is natural that the share of skilled workers needs to be increased. Moreover, sensitivity analysis in Subsection 2.4.2 shows that this parameter does not have a major role in influencing the most salient outcomes.

Parameters specific to the model are chosen to match the targeted moments. As I have 6 free parameters to match 5 moments, I impose an additional restriction on the efficiency of R&D labor in the potential entrants sector to be equal to the efficiency of the R&D labor in the incumbents sector, as there is no specific a priori reason why they should differ. Table 2.1 documents chosen parameter values and the justification for the choices. Note that in the Targeted section of the table the justification should be viewed as a joint system of conditions rather than a 1:1 correspondence between a specific parameter and targeted moment. Table 2.2 compares the model outcomes to targeted moments. Apart from the share of R&D expenditures in GDP, which is slightly lower in the model than in the data²², all moments are replicated satisfactorily.

The lower part of the Table 2.2 reports also the comparison of two non-targeted model

²¹Acemoglu et al. (2013) estimate their model using SMM and obtain point estimate of 7.8%. However, basing on other sources, they consider any number between 3.9% and 16.2% to be plausible.

²²It should be noted that the Gross Domestic Product: Research and Development by BEA contains all R&D expenditures performed in the US economy, and not only those performed by private businesses.

outcomes and their empirical counterparts. The “skilled wage premium” is obtained by calculating the ratio of weekly wages of supervisory employees to that of non-supervisory and production employees, which corresponds relatively well with the model object. However, as in the data around 17% of employees are considered “skilled” (versus 10% in the model), the empirical measure of “skilled wage premium” is very likely to be biased downwards.

Admittedly, the share of profits in GDP predicted by the model is lower than in the data, even if the model’s elasticity of substitution is relatively low compared to the usual parameter values. One reason for that may be that the model assumes that all establishments belong to a single industry, which lowers profitability compared to the situation of many industries with higher intra-industry but lower inter-industry elasticity of substitution.

Table 2.1: Calibration of the model parameters

Parameter	Description	Value	Justification
Fixed			
β	Discount factor	0.99	Standard (quarterly)
θ	Inverse of IES	2	Standard
κ	Inverse of Frisch elasticity	$1 - 2\theta$	Volatility of hours worked
σ	Elasticity of substitution	4	Christopoulou & Vermeulen (2010)
ρ_Z	Autocorr. of TFP process	0.95	Cooley & Prescott (1995)
σ_Z	Std. dev. of TFP shock	0.0055	Match std. dev. of output
s	Share of skilled workers	10%	Ballpark estimate
Targeted			
ι	Innovative step size	1.015	Annual pc. GDP growth
a	Incumbent R&D eff.	10	Expansions \approx contractions
a^e	Entrant R&D eff.	10	$a = a^e$
f	Incumbent labor req.	1	Share of R&D employment
f^e	Entrant labor req.	1	Share of R&D in GDP
δ^{exo}	Exog. exit shock prob.	0.02	Exit rate

Table 2.2: Long-run moments: comparison of model and data

Description	Model	Data	Source
Targeted			
Annual pc. GDP growth	2.02%	2.08%	BEA, 1948q1-2016q2
Relative share of expanding estabs.	1.00	1.01	BDM, 1992q3-2016q2
Exit rate ^a	3.07%	3.07%	BDM, 1992q3-2016q2
Share of R&D employment	0.97%	0.98%	NSF & CBP, 1964-2008
Share of R&D in GDP	2.07%	2.23%	BEA, 1948q1-2016q2
Non-targeted			
“Skilled wage premium”	2.59	2.09	BLS, 2006q1-2016q2
Share of profits in GDP	4.65%	6.53%	BEA, 1948q1-2016q2

^aCalculated from the data as the average between death and birth rates.

2.3.2 Model performance

As is standard in the business cycle literature, I present the comparison between the HP-filtered moments generated by the model and the data.

Data for the variables presented in the upper part of the table are based on the 1948q1-2016q2 sample. Output is based on Real Gross Domestic Product by BEA, Hours on Nonfarm Business Sector: Hours of All Persons²³ by BLS and Research and Development on appropriately deflated Gross Domestic Product: Research and Development by BEA.

Data for variables presented in the lower part of the table are based on the 1992q3-2016q2 sample, covering 97 periods, and come from the BDM. All variables before filtering were divided by the US labor force.

Table 2.3: Business cycle moments: comparison of model and data

Variable	Standard deviation		Correlation with Y		Autocorrelation	
	Data	Model	Data	Model	Data	Model
Output	1.58	1.58	1.00	1.00	0.82	0.77
Hours	1.36	0.73	0.86	0.99	0.89	0.72
R&D	2.36	2.95	0.32	0.98	0.89	0.68
Establishments	0.62	0.64	0.71	0.70	0.87	0.88
Expansions	2.84	1.22	0.82	0.89	0.75	0.91
Contractions	2.38	0.42	-0.11	-0.46	0.69	0.39
Net Entry	0.31	0.31	0.31	0.49	0.24	0.40

Model moments are based on 10000 simulated periods. Table 2.3 presents the set of moment statistics comparing model performance to data. Overall, the model is quite successful in capturing the cyclical characteristics of establishment dynamics.

The cyclical properties of the number of establishments are perfectly in line with the data. The model underpredicts the volatility of expansions and contractions, which may be caused by the fact that a significant share of US establishments does not adjust its employment quarter over quarter. It also underpredicts the volatility of hours worked, but this issue is common across the majority of business cycles literature. The model also slightly overpredicts the standard deviation of R&D expenditures, although as the next exercise shows, compared to R&D performed by private businesses, this statistic may well be too low.

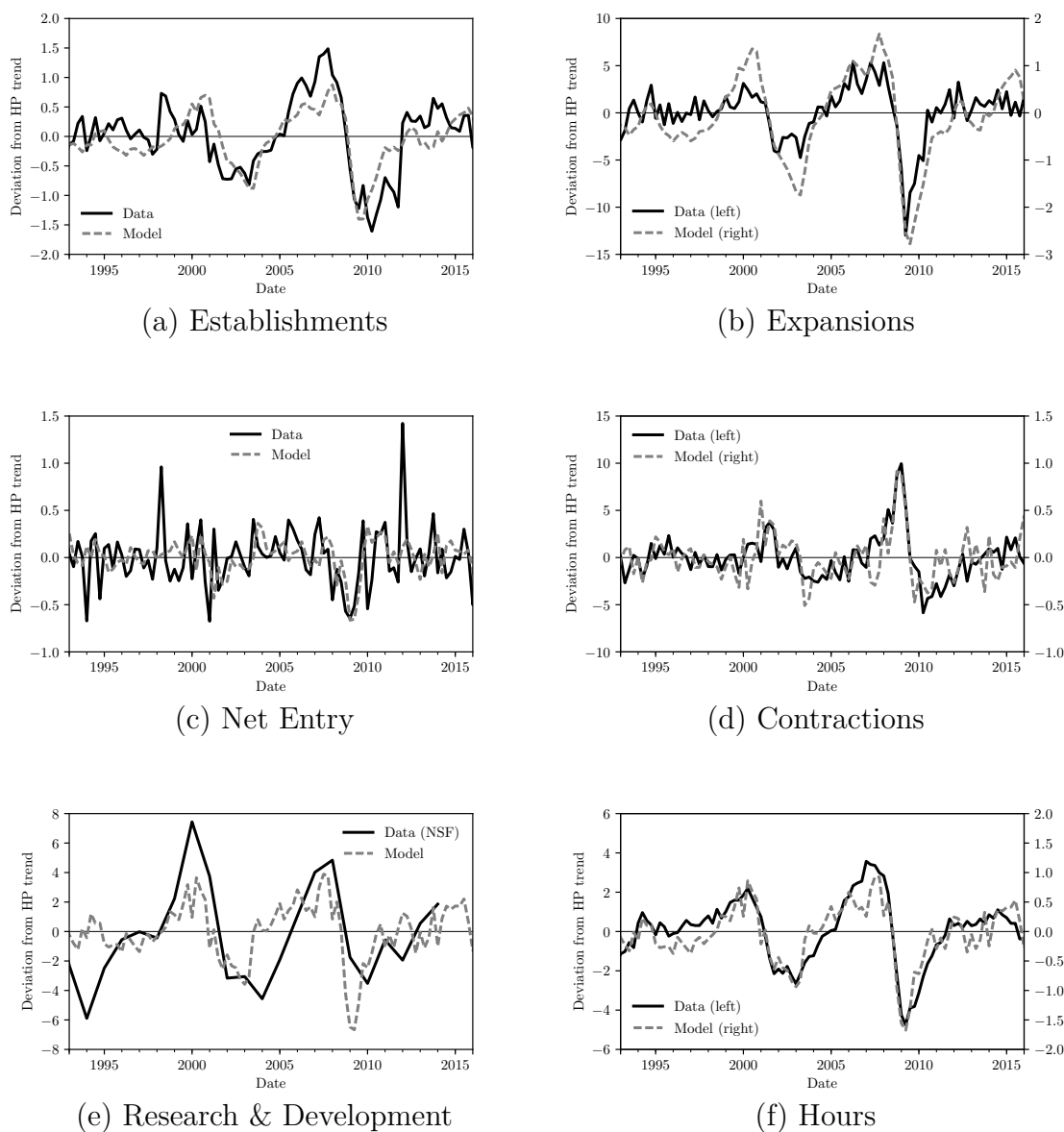
As an additional exercise to confirm that the model is able to replicate the comovements of variables in the data, I obtain the values of productivity shocks hitting the economy so that the HP-deviations of model output and US GDP are identical²⁴. In the next step I run the model conditional on this particular history of shocks and compare

²³Since in general the nonfarm business sector variables are more volatile than their economy-wide counterparts, the reported standard deviation of HP-deviations in hours is normalized by dividing by the ratio of volatility of Nonfarm Business Sector: Real Output by BLS and Real Gross Domestic Product.

²⁴In this step I construct a first-order approximation of the model and use Dynare version 4.4.3 to obtain the shock history via the estimation step.

the HP-deviations of model variables and their empirical counterparts. The results of this exercise are reported in Figure 2.1.

Figure 2.1: Hodrick-Prescott trend deviations: comparison of model and data (%)



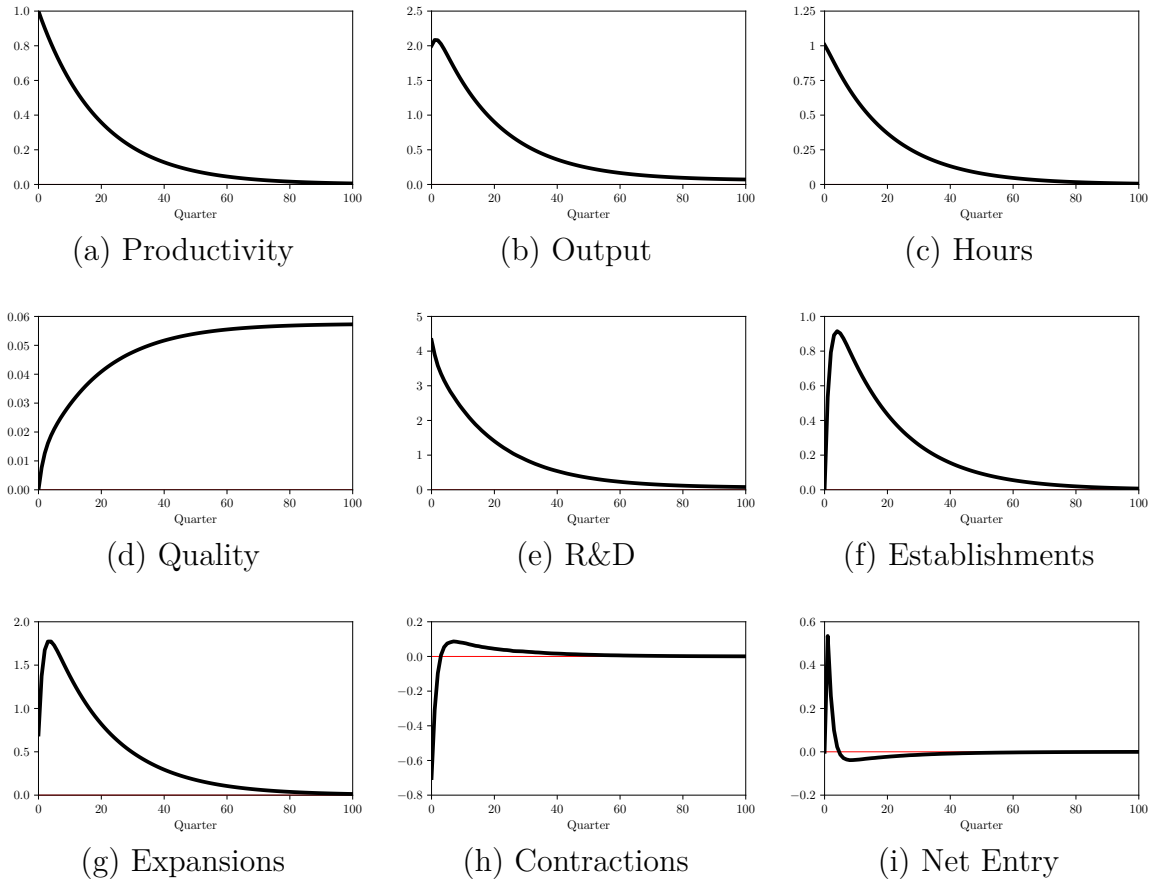
The performance of the model is very satisfactory as all the predicted movements match the data quite closely. The right hand column contains graphs where the volatility of the deviations is much bigger in the data than in the model and two scales are introduced. In the panel (e), instead of using the Research and Development series by BEA, I use the appropriately deflated Industrial R&D series by the National Science Foundation as it tracks specifically the R&D performed in the private business sector, although unfortunately is available only at the yearly frequency and with a significant lag. While the

model captures very well the dynamics of R&D in the vicinity of the dot-com bubble, it overpredicts the reduction in R&D at the beginning of the Great Recession.

2.3.3 Long shadows of temporary shocks

Figure 2.2 presents the impulse response function to a 1% productivity shock. A temporary increase in productivity (a) boosts output (b) directly, but also indirectly via an increase in hours worked (c) and the mass of active establishments (f), which gets bigger due to elevated net entry (i). The mass of expanding establishments (g) increases while the mass of contracting establishments (h) decreases on impact, while after a while increases due to bigger mass of active establishments. Last but not least, a positive shock to the goods producing sector increases R&D expenditures (e), which results in an increase of aggregate quality index (d) above its trend. Notice well that even if the shock eventually dissipates, the effects of the shock due to a level shift in quality, and in consequence the balanced growth path, remain. Almost 6% of the shock is translated to the level shift in the BGP, and around 2/3 of this effect is already in place 5 years after the shock.

Figure 2.2: Impulse response functions to 1% productivity shock (%)



2.4 Welfare analysis

The issue of quantifying welfare effects of business cycles has been recognized at least since Lucas (1987). His original assessment, basing on the variance of consumption, puts the cost of business cycles at roughly 1/20th of a percent of consumption. Although economic intuition suggests that those costs are probably much higher, the subsequent literature had limited success in generating large welfare effects of business cycles, see Lucas (2003) and Barlevy (2004).

When the aggregate quality process is endogenous, which gives rise to hysteresis, the welfare costs of business cycles are likely to be larger than in the case when aggregate quality follows an exogenous trend. Moreover, since the trend “productivity” growth can now be influenced, it naturally creates space for potentially welfare-improving policies.

2.4.1 Welfare cost of business cycles

The welfare cost of business cycles can be readily assessed using the consumption equivalent transformation. It denotes a lifetime percentage change in the consumption path of an agent which makes her indifferent across living in two distinct states of the world. The equivalent, denoted with eq , can be computed via the following procedure:

$$W(eq) = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{((1+eq)c_t)^{1-\theta}}{1-\theta} + u_t^n \right] = (1+eq)^{1-\theta} U_0^c + U_0^n = U_0^{BGP}$$

$$eq = \left(\frac{U_0^{BGP} - U_0^n}{U_0^c} \right)^{\frac{1}{1-\theta}} - 1$$

where u_t^n denotes the instantaneous disutility of labor, and U_0^c and U_0^n denote expected lifetime utility from consumption and expected lifetime disutility of labor, respectively.

Table 2.4 presents the comparison of expected welfare in three states of the world: non-stochastic (BGP) and two stochastic: one in which growth is exogenous, i.e. where the R&D activity is not sensitive to the shocks, and the one with the endogenous growth subject to business cycles mechanism. In line with the literature, the cost of business cycles, expressed in consumption equivalent terms, is very small if the growth rate is exogenous, since the path of output returns to the balanced growth path reasonably quickly. However, if endogenous growth rates react to shocks, generating level shifts in the Balanced Growth Path, the expected output path becomes much more volatile and can deviate from the BGP significantly. This increased uncertainty in outcomes translates to an almost 100-fold increase in the consumption equivalent, making the number two orders of magnitude higher than in the exogenous growth case.

Table 2.4: Welfare cost of business cycles

State of the world	Welfare	U^c	U^n	Consumption equivalent
Non-stochastic (BGP)	-122.26	-168.06	45.80	–
Stochastic with exogenous growth	-122.37	-168.19	45.83	0.06%
Stochastic with endogenous growth	-129.89	-175.73	45.83	4.54%

2.4.2 Sensitivity analysis

Before I analyze the effects of policies, I discuss how the key model statistics – aggregate growth rate, the level shift in the BGP after 5 and 25 years, and the expected social welfare – depend on the model parameters. For most of the parameters I will consider a 10% change in a parameter value. The exception to this operation are the discount factor β , where I increase it from 0.99 to 0.995 and productivity shock autocorrelation ρ_Z which is increased from 0.95 to 0.97. Table 2.5 documents the effects of the changes in parameter values on the aggregate outcomes and welfare.

Both the growth rate along the BGP, as well as in the stochastic equilibrium, are higher when: innovative step is larger, incumbents' R&D efficiency is higher, incumbents' fixed cost is higher, exogenous exit shock probability is higher, and households are more patient. The reason why an economy with higher incumbents' fixed cost grows faster stems from the fact that it is populated by fewer establishments and each of them can employ more R&D labor. An economy with higher exit shock probability grows faster since the mass of entrants, who perform more radical innovations than the incumbents, is higher. The economy grows slower when the elasticity of substitution is higher, as the gains from innovation are smaller.

The shift in level of BGP due to a shock is higher when: innovative step is larger, entrants' R&D efficiency is higher, exogenous exit shock probability is higher, both the productivity autocorrelation and volatility are higher, and households are more patient and more risk-averse. Again, higher exit shock probability results in higher population of entrants, and a higher entry rate in response to a positive shock. In this model, higher risk aversion amplifies the reaction in skilled labor, which results in stronger reaction of aggregate quality. The shift in level of BGP is smaller when incumbents' R&D efficiency is higher, fixed costs are higher, and elasticity of substitution is higher. Higher R&D efficiency of incumbents increases their share of R&D employment at the expense of entrants who have innovative advantage, thus dampening the effect of a shock.

As it turns out, the share of skilled workers in the economy does not influence the aggregate growth rate nor the cyclical properties of the model, although it affects the population of active establishments in the economy.

When discussing welfare issues, I exclude the parameters that directly influence the social welfare function, as comparing across different utility functions is infeasible. The last column of Table 2.5 includes the consumption equivalent between the world with a changed parameter and the baseline world. Slightly counter-intuitively, positive consumption equivalent signals welfare deterioration. A general pattern that emerges from comparing aggregate outcomes and changes in the welfare is that the larger the shifts

in the BGP, the lower the welfare, even if the aggregate growth rates are higher. This underscores the importance of the welfare costs stemming from living in a fundamentally more uncertain world, compared to the one where “productivity” reverts to an exogenous trend.

Accordingly, higher innovative step size and higher standard deviation of shocks are strongly welfare deteriorating. On the other hand, higher entry barriers also have a slightly negative effect on welfare, but this effects stems from slightly lower aggregate growth rates, rather than from the volatility channel. Higher probability of an exit shock, higher elasticity of substitution, higher incumbent’s fixed costs and higher R&D efficiency all bring welfare improvements. Usually those improvements are a result of lowered volatility of the economy, although in the case of exit shock probability an increase in aggregate growth rates dominates the effect of slightly elevated volatility.

Two conclusions can be drawn from this exercise. First, the quantitative impact of a shock on the shift in BGP is relatively robust with respect to changes in specific parameters. The parameters which have the strongest impact on the quantitative effect – elasticity of substitution, productivity process autocorrelation and elasticity of intertemporal substitution – were all taken from previous literature and did not depend on the specifics of calibration. Second, there is ample scope for policy intervention as policies limiting the economy’s volatility are expected to generate nontrivial welfare effects.

Table 2.5: Sensitivity analysis

	γ^{BGP}	γ	ΔQ_{20}	ΔQ_{100}	U^{BGP}	U	U^c	U^n	eq^a
Baseline	2.02	2.04	2.25	3.15	-122.26	-129.89	-168.06	45.80	–
ι	2.16	2.19	2.33	3.28	-127.88	-136.22	-184.22	48.00	3.56%
a	2.06	2.09	2.17	3.03	-120.87	-128.04	-173.35	45.31	-1.06%
a^e	2.02	2.04	2.26	3.16	-121.52	-129.11	-174.67	45.55	-0.45%
f	2.06	2.08	2.21	3.07	-119.78	-126.92	-171.82	44.90	-1.70%
f^e	2.02	2.04	2.22	3.12	-122.67	-130.29	-176.28	45.99	0.22%
δ^{exo}	2.09	2.11	2.28	3.20	-115.34	-122.31	-165.46	43.15	-4.38%
σ	1.68	1.71	1.86	2.64	-119.14	-126.01	-173.87	47.86	-2.19%
$\rho_Z = 0.97$	2.02	2.04	2.24	4.03	-122.26	-129.85	-175.71	45.86	-0.03%
σ_Z	2.02	2.05	2.48	3.47	-122.26	-131.50	-177.34	45.84	0.91%
$\beta = 0.995$	2.08	2.11	2.29	4.14					
θ	2.00	2.04	3.66	6.61					
κ	2.02	2.04	1.79	3.25					
s	2.02	2.04	2.25	3.15					

^aThe lower the number, the higher the welfare gain

2.4.3 Policy experiments

The conclusions of the previous sections also have implications for the optimal industrial policy. In particular, encouraging entry by either lowering entry barriers or by subsidizing entrants’ R&D yields positive welfare effects in the stochastic equilibrium. Additionally, direct subsidies to incumbents are welfare deteriorating, while R&D subsidies boost welfare.

The above results fall in line with those obtained by the endogenous growth literature in deterministic settings²⁵. The value added of my approach is in considering the effects of countercyclical subsidies. Such subsidies do not affect significantly the aggregate growth rates directly, but can act as moderators of business cycle fluctuations and via the volatility channel affect welfare.

As neither the productivity shock nor the aggregate quality level are observable, I set up a scheme where subsidy reacts to variables that are observable almost in real time: the level of output Y and mass of active establishments M . Both of those schemes yield similar outcomes²⁶. The subsidy/tax is financed by lump sum taxes/transfers levied on the households. As the fixed and R&D costs are in effect incomes of skilled workers, the subsidy/tax does not redirect the resources from the private economy, and merely affects the incentives of entrants and incumbents. Therefore, one must take the results with a grain of salt as implementable tax/subsidy schemes would introduce additional distortions.

Table 2.6 documents the effects of applying countercyclical subsidies to the model economy. A symmetric subsidy/tax lowers the effective costs of skilled labor in the recession and increases those costs during expansions, therefore counteracting the cyclical effects on the skilled workers' wage. Similarly as in the constant subsidy case, subsidizing either entrants' R&D efforts or lowering the costs of entry generates positive welfare effects.

The situation is however radically different in the case of incumbents: while it is welfare improving to implement a constant subsidy to incumbents' R&D efforts, countercyclical subsidies are welfare deteriorating. This effects is the result of competition for skilled labor between entrants and incumbents: subsidy to incumbents' R&D diverts resources from entrants where they are needed the most: in times when entry is already depressed. This is also evidenced by increased impact of shock on the level shift in BGP compared to the baseline.

On the other hand, countercyclical subsidies to the fixed cost of incumbents increase welfare, whereas constant subsidies were welfare deteriorating. The reason is that while permanent subsidies discourage entry, maintain lower productivity establishments and thus divert resources from more efficient uses, countercyclical subsidies prevent inefficient exits and thus reduce significantly the volatility of the economy. In fact, those subsidies generate the strongest positive welfare effects. This conclusion, if supported by further studies, gives justification for policies designed to support existing establishments in the severe downturns.

²⁵See e.g. Acemoglu et al. (2013).

²⁶Since the volatility of establishments is lower than that of output, it is natural that the quantitative estimates will be lower in the case of establishment subsidy.

Table 2.6: Effects of countercyclical subsidies

	ΔQ_{20}	ΔQ_{100}	U	U^c	U^n	μ^a
Baseline	2.25	3.15	-129.89	-175.73	45.83	—
0.5% subsidy if Y is 1% below trend						
f	0.91	1.22	-128.84	-174.66	45.82	-0.60%
a	3.03	4.42	-133.16	-179.00	45.84	1.86%
f^e	2.14	2.99	-129.67	-175.51	45.83	-0.13%
a^e	2.22	3.10	-129.83	-175.66	45.83	-0.04%
0.5% subsidy if M is 1% below trend						
f	1.69	2.33	-129.64	-175.47	45.83	-0.14%
a	2.58	3.71	-130.99	-176.82	45.84	0.62%
f^e	2.24	3.12	-129.74	-175.57	45.83	-0.09%
a^e	2.25	3.15	-129.85	-175.68	45.83	-0.03%

^aThe lower the number, the higher the welfare gain

2.5 Conclusions

As documented by Comin & Gertler (2006), Barlevy (2007) and Anzoategui et al. (2016), expenditure on R&D is volatile and procyclical. In this Chapter I have presented an endogenous growth model, featuring monopolistically competitive, heterogeneous establishments that endogenously decide on the intensity of R&D. The model is consistent with the above-mentioned facts and generates predictions on the strength of the impact of business cycle fluctuations on the endogenous growth rates of the economy.

The results suggest that the mechanism governing innovation dynamics generates hysteresis effects of temporary shocks on the BGP level, translating almost 6% of the strength of a shock to the level shift of the BGP. This observation, coupled with other “missing generation of firms” effects, as identified by Siemer (2014) and Messer et al. (2016), urges to reassess the previous estimates of the welfare costs of business cycles.

I find that the welfare effects of business cycles are nontrivial and of two orders of magnitude higher than in the models with exogenous growth. Considerable welfare effects and the potential to influence endogenous growth rates makes scope for policy intervention. In line with existing endogenous growth literature, e.g. Acemoglu et al. (2013), I find that subsidizing incumbents is welfare deteriorating, while subsidizing entry and R&D investments is welfare improving. These results do not carry over to the case of countercyclical subsidies, where I find that subsidizing incumbents’ R&D expenditures is strongly welfare deteriorating, while subsidizing incumbents’ operation costs becomes welfare enhancing. This conclusion, if supported by further studies, gives justification for policies designed to support existing establishments in the severe downturns.

Chapter 3

Innovation and endogenous growth over business cycle with frictional labor markets

The issue of interactions between endogenous growth rates and unemployment dynamics has been analyzed at least since Aghion & Howitt (1994). To contribute to this literature, I extend the model from Chapter 2 by including a frictional labor market, subject to the search and matching friction in the tradition of Diamond (1982) and Mortensen & Pissarides (1994). I follow an approach proposed by Gertler & Trigari (2009) that assumes nonlinear vacancy posting costs and is remarkably successful in replicating the labor market dynamics. Therefore this Chapter is also related to the literature focusing on the impact of labor market frictions, such as the presence and level of firing costs, on reallocation and productivity growth. In a seminal paper Hopenhayn & Rogerson (1993) assess the impact of firing costs on reallocation and productivity, and find non-negligible negative effects. Similar conclusions are reached by the works reviewed and systematized in Hopenhayn (2014). Bassanini et al. (2009) find that firing costs tend to reduce growth in industries where firing costs are more likely to be binding. Davis & Haltiwanger (2014) argue that a recent decrease in labor market fluidity in the United States negatively impacted job reallocation rates and harmed productivity growth. Da-Rocha et al. (2016) find much bigger static and dynamic losses in aggregate total factor productivity when the presence of firing costs alters the establishment-level productivity distribution. Mukoyama & Osotimehin (2017) analyzes the effects of firing taxes in a model with rich firm dynamics, although the model does not incorporate aggregate shocks. Although the analysis of the impact of firing costs is not possible in the setup chosen for this Chapter, the fluidity of the labor market is affected by the level of hiring costs, and some parallel conclusions can be drawn.

The Chapter is organized as follows. The next section describes the model, deriving the problem of incumbents and potential entrants, and describing the details of labor market frictions. The second section discusses the data sources and parameter values, including those that are estimated. This section also documents stochastic properties of the model economy in comparison to the data. The third section is devoted to a discussion of policy implications, providing an estimate of the welfare cost of business cycles for the US economy and a comparison of the effects of several subsidy schemes.

The last section concludes.

3.1 Model

Compared to the previous Chapter, the model presented here features two major changes. First, I introduce physical capital as another factor of production. Second, instead of modeling the labor market as Walrasian, I assume that labor market is subject to the search and matching friction as in Gertler & Trigari (2009). Following Christiano et al. (2011) I assume that the hiring and wage bargaining processes are managed by employment agencies who then supply firms with labor services at a common price.

3.1.1 Households

There is a unit mass of representative households. Each representative household consists of a large family of workers, giving rise to within-household insurance, as in Merz (1995) and Andolfatto (1996). Any individual worker may be within a given time period employed and receiving wage income or unemployed and receiving unemployment benefits¹. As in Acemoglu et al. (2013), there are two types of workers: skilled of mass s and unskilled of mass $1 - s$. Regardless of the labor market status or skill category each individual enjoys the same level of consumption.

The representative household aims to maximize expected lifetime utility of its members:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\theta}}{1-\theta} \quad (3.1)$$

where β is the discount factor c_t denotes per capita consumption and θ is the inverse of the elasticity of intertemporal substitution. The household is subject to the following budget constraint:

$$c_t + k_{t+1} = (1 + r_t - dp) k_t + s [w_t^s n_t^s + b_t^s (1 - n_t^s)] + (1 - s) [w_t^u n_t^u + b_t^u (1 - n_t^u)] + t_t$$

where k_t denotes per capita stock of physical capital which yields interest rate r_t , dp is the rate of capital depreciation, w_t^s and w_t^u are real wage rates for skilled and unskilled labor, respectively, n_t^s and n_t^u are the shares of skilled and unskilled workers that are currently employed, b_t^s and b_t^u denote unemployment benefits, and t_t denotes any lump sum net transfers that households receive, including all profits.

The first order conditions of the households result in the following Euler equation:

$$c_t^{-\theta} = E_t \left[\beta c_{t+1}^{-\theta} (1 + r_{t+1} - dp) \right] \quad (3.2)$$

As all firms in the economy are ultimately owned by households, I assume that their managers discount future profit streams consistent with the stochastic discounting kernel of the households:

$$\Lambda_{t,t+1} = E_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\theta} \right] \quad (3.3)$$

¹Note that I abstract from the real-world possibility that an individual is not active on the labor market.

3.1.2 Final goods producer

The final goods producing sector is modeled as a single representative perfectly competitive firm that transforms a continuum of mass $M_t \in (0, 1)$ of intermediate good varieties² into final goods using the CES aggregator:

$$Y_t = \left[\int_0^{M_t} y_t(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}$$

where $y_t(i)$ denotes the output of i -th variety and $\sigma \in (1, \infty)$ is the elasticity of substitution between any two varieties. The standard solution of the cost minimization problem yields the price index of the final good as a function of the varieties' prices $P_t(i)$:

$$P_t = \left[\int_0^{M_t} P_t(i)^{1-\sigma} di \right]^{\frac{1}{1-\sigma}}$$

as well as the Hicksian demand function for the i -th variety:

$$y_t(i) = Y_t p_t(i)^{-\sigma} \quad (3.4)$$

where $p_t(i) = P_t(i) / P_t$ is the variety's price relative to the price index.

3.1.3 Intermediate goods producers

The intermediate goods producing sector is modeled as a single industry sector populated by monopolistically competitive continuum of mass M_t of active establishments, each producing a distinct variety. To produce, an establishment needs to incur fixed costs f_t , representing expenditures on management and other non-production activities. The production function of an establishment is of a Cobb-Douglas functional form:

$$y_t(i) = Z_t k_t^p(i)^\alpha [q_t(i) n_t^p(i)]^{1-\alpha}$$

where Z_t is the stochastic aggregate productivity parameter, $k_t^p(i)$ and $n_t^p(i)$ denote, respectively, the employment of capital services and unskilled labor, $q_t(i)$ is the quality level of i -th variety at time period t , and α is the elasticity of output with respect to capital.

The solution of the cost minimization problem yields the following expression for the marginal cost, depending on the idiosyncratic quality level of an establishment:

$$mc_t^p(i) = \frac{1}{Z_t} \left(\frac{r_t}{\alpha} \right)^\alpha \left(\frac{\tilde{w}_t^u / q_t(i)}{1 - \alpha} \right)^{1-\alpha}$$

where \tilde{w}_t^u denotes the unskilled wage paid to the employment agency.

It is straightforward to show that the optimal pricing strategy given flexible prices and the demand for an individual variety given by Equation 3.4 follows the standard constant mark-up pricing formula:

$$p_t(i) = \frac{\sigma}{\sigma - 1} mc_t(i)$$

²The condition that the mass of intermediate goods varieties is bounded between 0 and 1 is supported by assuming that each individual possesses an idea for a product, but only a subset of those individuals are entrepreneurs and only a fraction of possible goods is actively produced.

Following Melitz (2003), I assume that the distribution of idiosyncratic quality levels at time t is described by some probability density function $\mu_t(q)$ with support on a subset of $(0, \infty)$. It is convenient to define an aggregate quality index Q_t such that the aggregate state of the intermediate goods producing sector can be summarized as if it was populated by mass M_t of establishments all with quality level Q_t . The index is given by the following formula:

$$Q_t^{1-\alpha} = \left[\int_0^\infty (q^{1-\alpha})^{\sigma-1} \mu_t(q) dq \right]^{\frac{1}{\sigma-1}}$$

As the aggregate quality level grows over time, the idiosyncratic quality levels of individual establishments are best expressed in relative terms. Therefore, I construct the following measure of relative quality:

$$\phi_t(i) \equiv (q_t(i) / Q_t)^{(1-\alpha)(\sigma-1)}$$

The aggregate final goods output can be then expressed as:

$$Y_t = M_t^{\frac{1}{\sigma-1}} Z_t (K_t^p)^\alpha (Q_t N_t^p)^{1-\alpha} \quad (3.5)$$

where K_t^p and N_t^p denote, respectively, aggregate capital stock and employment in the production sector and the dependence of output on M_t reflects the love-for-variety phenomenon.

3.1.4 Incumbents

I assume that each incumbent establishment can direct resources to R&D activities in attempt to improve their varieties' quality. The success probability function is taken from Pakes & McGuire (1994) and Ericson & Pakes (1995):

$$\chi_t(i) = \frac{ax_t(i)}{1 + ax_t(i)}$$

where $\chi_t(i)$ denotes the probability of making a quality improvement and a is a parameter that describes the efficiency of R&D input $x_t(i)$ in generating improvements. R&D input requires a combination of skilled labor and capital:

$$x_t(i) = \frac{k_t^x(i)^\alpha [Q_t n_t^x(i)]^{1-\alpha}}{Q_t \phi_t(i)}$$

where $k_t^x(i)$ and $n_t^x(i)$ denote, respectively, the employment of capital services and skilled labor.

The presence of aggregate and relative quality levels in the expression lends itself to an intuitive interpretation. Aggregate quality level in the numerator multiplies with R&D laborers as they have access to a pool of common knowledge. However, over time it is harder to come up with new ideas unless more resources are committed to R&D activities, which is captured by aggregate quality level in the denominator. Finally, the presence of relative quality level in the denominator represents the catch-up and headwind effects, depending on establishments' position in the quality distribution.

The solution of the cost minimization problem results in the following expression for the marginal cost in the R&D sector:

$$mc_t^x(i) = Q_t^\alpha \left(\frac{r_t}{\alpha}\right)^\alpha \left(\frac{\tilde{w}_t^s}{1-\alpha}\right)^{1-\alpha} \phi_t(i) \equiv \bar{m}c_t^x \phi_t(i)$$

where \tilde{w}_t^s denotes the skilled wage paid to the employment agency, and $\bar{m}c_t^x$ is the skilled marginal cost component common to all establishments.

I also assume that the managerial activities require the same combination of physical capital and skilled labor as R&D activities. Therefore, the fixed cost can be expressed as a product of the common skilled marginal cost and a constant f . Accordingly, the real profit can be expressed as the following function, which is affine in terms of $\phi_t(i)$:

$$\pi_t(i) = Y_t \left[\left(\frac{1}{\sigma M_t} - \frac{\omega_t}{a} \frac{\chi_t(i)}{1-\chi_t(i)} \right) \phi_t(i) - \omega_t f \right] \quad (3.6)$$

and where $\omega_t \equiv \bar{m}c_t^x/Y_t$ is the ratio of common skilled marginal cost and aggregate output.

The dynamic problem of the incumbents can be cast in the recursive form. Since all establishments with the same relative quality levels will make identical decisions, I drop the subscript i . Additionally, for establishments with low enough ϕ_t the expected stream of future profits turns negative and they decide to exit at the end of the current period.

The value of an establishment with relative quality level ϕ_t is given by the following expression:

$$V_t(\phi_t) = \max_{\chi_t \in [0,1]} \{ \pi_t(\phi_t, \chi_t) + \max \{0, E_t[\beta \Lambda_{t,t+1} (1-\delta_t) V_{t+1}(\phi_{t+1} | \phi_t, \chi_t)] \} \}$$

where $\Lambda_{t,t+1}$ is the stochastic discount factor consistent with the households' valuation of current and future marginal utility from consumption (Equation 3.3), δ_t denotes endogenous establishment death shock probability, which will be described in detail later, and the relative quality of a variety in the next period is subject to the following lottery³:

$$\phi_{t+1} = \begin{cases} \iota \phi_t / \eta_t & \text{with probability } \chi_t \\ \phi_t / \eta_t & \text{with probability } 1 - \chi_t \end{cases}$$

where ι denotes the size of the innovative step and η_t is the rate of growth of the aggregate quality index (raised to a certain power), taken as given by the individual establishments:

$$\eta_t \equiv \left(\frac{Q_{t+1}}{Q_t} \right)^{(1-\alpha)(\sigma-1)}$$

Since the aggregate quality index is trending upwards over time, it is useful to consider the following stationarization. Define $v_t(\phi_t) \equiv V_t(\phi_t)/Y_t$ to be the ratio of the value

³The underlying absolute quality levels evolve according to the lottery:

$$q_{t+1} = \begin{cases} \iota^{1/[(1-\alpha)(\sigma-1)]} q_t & \text{with probability } \chi_t \\ q_t & \text{with probability } 1 - \chi_t \end{cases}$$

function and current aggregate output. For the problem rewritten in relative terms the level of aggregate quality becomes irrelevant, and its rate of growth is a function of the current state only.

Moreover, for large enough ϕ_t the probability that an establishment will want to exit in the foreseeable future is very small, and the $\max\{0, \cdot\}$ operator can be disregarded. As the real profit function is affine in ϕ_t and the value function is a weighted sum of present and future profit streams, it is also affine in ϕ_t . Therefore, I impose the affine functional form on $v_t(\phi_t) \equiv A_t + B_t\phi_t$:

$$A_t + B_t\phi_t = \max_{\chi_t \in [0,1)} \left\{ \left(\frac{1}{\sigma M_t} - \frac{\omega_t}{a} \frac{\chi_t}{1-\chi_t} \right) \phi_t - \omega_t f + E_t \left[\beta \Lambda_{t,t+1} (1 - \delta_t) \left(\frac{Y_{t+1}}{Y_t} \right) (A_{t+1} + B_{t+1}\phi_{t+1}) \right] \right\} \quad (3.7)$$

The solution to the incumbents' problem must then satisfy the following first order and envelope conditions:

$$0 = -\frac{\omega_t}{a} \frac{1}{(1 - \chi_t)^2} + E_t \left[\beta \Lambda_{t,t+1} (1 - \delta_t) \left(\frac{Y_{t+1}}{Y_t} \right) \left(B_{t+1} \frac{(\iota - 1)}{\eta_t} \right) \right] \quad (3.8)$$

$$B_t = \left(\frac{1}{\sigma M_t} - \frac{\omega_t}{a} \frac{\chi_t}{1 - \chi_t} \right) + E_t \left[\beta \Lambda_{t,t+1} (1 - \delta_t) \left(\frac{Y_{t+1}}{Y_t} \right) B_{t+1} \frac{\chi_t (\iota - 1) + 1}{\eta_t} \right] \quad (3.9)$$

Note that the relative quality level does not impact the optimal innovative success probability χ_t , as long as ϕ_t is high enough, in line with Gibrat's law.

Obviously, one needs to specify the decisions of establishments with lower levels of ϕ_t . For sufficiently low levels of ϕ_t the establishment exits and thus does not engage in R&D activities at all. Therefore, its value function is given by:

$$A_t + B_t\phi_t = \frac{1}{\sigma M_t} \phi_t - \omega_t f \quad (3.10)$$

It now remains to specify what happens in the intermediate range of relative quality levels. For the sake of tractability I opt to represent the true value function with its piecewise linear approximation, namely, I extend the functions given by Equations 3.7 and 3.10 until they intersect for the relative quality level ϕ_t^* , given implicitly by the following condition:

$$\frac{\omega_t}{a} \frac{\chi_t}{1 - \chi_t} \phi_t^* = E_t \left[\beta \Lambda_{t,t+1} (1 - \delta_t) \left(\frac{Y_{t+1}}{Y_t} \right) \left(A_{t+1} + B_{t+1} \frac{\chi_t (\iota - 1) + 1}{\eta_t} \phi_t^* \right) \right] \quad (3.11)$$

All establishments with relative quality levels no higher than ϕ_t^* exit, and all continuators choose the same level of χ_t . By assuming that the quality is distributed according to the Pareto distribution with power parameter equal to one⁴, I am able to provide a closed form expression for the mass of establishment exits:

$$M_t^x = M_t (1 - \chi_{t-1}) \left(1 - \frac{\phi_{t-1}^*}{\phi_t^* \eta_{t-1}} \right) \quad (3.12)$$

⁴This assumption is ubiquitous in the firm size distribution literature. For empirical support see e.g. Axtell (2001).

3.1.5 Entrants

The mass of prospective entrants is assumed to be a priori unbounded. Similar to active establishments, they can engage in R&D activities. In contrast to incumbents, the successful outcome of their innovation effort is not an improvement in an existing product, but rather creating a new one, which may or may not replace an existing variety.

To attempt entry, prospective entrants hire physical capital and skilled labor just as incumbents do, including also the necessity to cover fixed costs. Successful entrants begin their production in the next period. The stationarized expected value of entry is given by:

$$v_t^e = \max_{\chi_t^e \in [0,1]} \left\{ -\omega_t \left(f^e + \frac{1}{a^e} \frac{\chi_t^e}{1 - \chi_t^e} \right) + \chi_t^e \text{E}_t \left[\Lambda_{t,t+1} \left(\frac{Y_{t+1}}{Y_t} \right) v_{t+1}(\phi_{t+1}^e) \right] \right\} \quad (3.13)$$

where χ_t^e is the probability of entering the market next period, a^e is a parameter that describes the efficiency of R&D input and ϕ_{t+1}^e denotes the relative quality draw upon entry. Since entrants tend to perform more radical innovations than incumbents, as emphasized by e. g. Acemoglu & Cao (2015) and Garcia-Macia et al. (2016), I assume that they draw from the incumbents' distribution of quality levels, upscaled by a factor which precludes the need to resort to limit pricing.

The first order condition of the entrants' problem can be expressed as:

$$\frac{\omega_t}{a^e} \frac{1}{(1 - \chi_t^e)^2} = \text{E}_t \left[\Lambda_{t,t+1} \left(\frac{Y_{t+1}}{Y_t} \right) v_{t+1}(\phi_{t+1}^e) \right] \quad (3.14)$$

Additionally, since the mass of prospective entrants is unbounded, the following free entry condition holds in every period⁵:

$$v_t^e = 0 \quad (3.15)$$

Hence, if the mass of successful entrants is denoted by M_t^e and the chosen success probability is χ_t^e , then the mass of agents attempting entry has to equal M_t^e / χ_t^e .

The modeled entry process is undirected, but allows for the possibility that a successful entrant leapfrogs over an incumbent. To capture this feature, I assume that the space of all possible varieties occupies a unit interval, while active establishments occupy its subset M_t . This represents the notion that every individual in an economy possesses a potential business idea but only a fraction of them become entrepreneurs and their varieties are produced. Since the mass of households equals unity, it is natural to assume that the mass of potential ideas also equals unity.

Upon successful entry the new establishment randomly draws its "location" from the unit interval and a fraction of them replaces active establishments. To ensure no limit pricing in equilibrium, I assume that entrants enjoy a relative quality advantage over the incumbents. Therefore, if an entrant replaces an incumbent that has innovated successfully, the product line will be characterized by $[\iota\sigma / (\sigma - 1)]^{1/[(1-\alpha)(\sigma-1)]}$ times higher quality level than in the previous period, and in case of replacing an incumbent that has not succeeded in innovating the product line's quality increases by a factor of

⁵The model in the previous Chapter admitted the possibility of no entry. However, the model in this Chapter is solved using perturbation methods which do not allow for inequality constraints.

$[\sigma/(\sigma-1)]^{1/[(1-\alpha)(\sigma-1)]}$. Accordingly, the expected relative quality level of entrants is equal to:

$$E_t [\phi_{t+1}^e] = \frac{\sigma}{\sigma-1}$$

I can now specify the process for the endogenous probability of an incumbent receiving an exit shock. There are three conditions under which an active establishment exits, and I assume that at the end of each period the events follow a specific order. First, the incumbents with relative quality level below ϕ_t^* exit “voluntarily” as their varieties become obsolete. Second, incumbents receive exogenous exit shocks. Finally, a fraction of incumbents are leapfrogged by entrants and thus creatively destroyed. Therefore, the mass of active establishments in the next period is given by:

$$M_{t+1} = M_t - M_t^x - \delta^{exo} (M_t - M_t^x) + [1 - (1 - \delta^{exo}) (M_t - M_t^x)] M_t^e$$

where δ^{exo} is the exogenous exit shock probability and the mass of successful entrants M_t^e is multiplied by the probability that an entrant draws an “unoccupied” location. As by definition creative destruction replaces an incumbent with an entrant, it does not directly affect the mass of active establishments. The expression for active establishment mass can be also written as:

$$M_{t+1} = M_t - M_t^x - \delta_t (M_t - M_t^x) + M_t^e \quad (3.16)$$

Then by comparing the two formulations one gets the following expression for endogenous exit shock probability:

$$\delta_t = 1 - (1 - \delta^{exo}) (1 - M_t^e) \quad (3.17)$$

Intuitively, the probability of not receiving an exit shock is a product of the probabilities of not receiving an exogenous shock and not being creatively destroyed, as the two are independent from each other.

It is now possible to characterize the process governing the evolution of the aggregate quality index. First, by the law of large numbers, a fraction χ_t of incumbents with relative quality levels above ϕ_t^* manage to improve their varieties, while the incumbents with obsolete varieties exit. Second, incumbents receive death shocks which are uncorrelated with their quality levels and thus leave the distribution unchanged. Finally, entrants draw their quality from the distribution of incumbents’ qualities, rescaled upwards. By assuming Pareto distribution of quality levels it is possible to derive the exact closed form expression for the rate of growth of the aggregate quality index:

$$\eta_t = (1 - \chi_t + \chi_t \iota) \left(1 - \frac{M_t^e}{M_{t+1}} + \frac{M_t^e}{M_{t+1}} \frac{\sigma}{\sigma-1} \right) \quad (3.18)$$

3.1.6 Frictional labor markets

I assume that labor markets are subject to the search and matching friction. At the end of each period a constant fraction of workers randomly separates from their previously held job positions and enter the pool of unemployed. The transition from the unemployed to employed state depends on the endogenously determined job finding probability, which

is influenced by the intensity of hiring. The assumption of constant separation rate and fluctuating hiring rate is consistent with the US data, as argued by Shimer (2005, 2012), although Petrongolo & Pissarides (2008) point out that it might not be an appropriate assumption for other countries.

I also assume that the unskilled and skilled labor markets are separated, with differing unemployment rates, vacancy rates, and so on. To facilitate exposition, and since both markets operate based on the same principles, I present the workings of the representative labor market, omitting the superscript.

3.1.6.1 Aggregate labor market dynamics

By excluding the possibility that an agent can be inactive on the labor market, the mass of unemployed workers is given by:

$$u_t = 1 - n_t \quad (3.19)$$

The mass of new matches m_t is a function of the mass of unemployed and the aggregate mass of vacancies v_t :

$$m_t = \sigma_m u_t^\psi v_t^{1-\psi} \quad (3.20)$$

where the parameter σ_m describes the efficiency of the matching process and ψ is the elasticity of matches with respect to the mass of unemployed.

The job finding probability p_t and job filling probability q_t can be obtained via the following transformation:

$$p_t = m_t / u_t \quad (3.21)$$

$$q_t = m_t / v_t \quad (3.22)$$

Following Gertler & Trigari (2009) and Gertler et al. (2008), and in contrast to the standard modeling approach by Mortensen & Pissarides (1994), I assume convex costs with respect to the hiring rate⁶:

$$x_t = \frac{q_t v_t}{n_t} \quad (3.23)$$

The process for mass of employed workers is given by the following relationship:

$$n_{t+1} = (\rho + x_t) n_t \quad (3.24)$$

where $1 - \rho$ is a constant separation rate.

3.1.6.2 Employment agencies and workers

Since the problem of the individual establishments is already quite complex and adding idiosyncratic employment and wage levels would make the model intractable, I follow Christiano et al. (2011) in assuming that both hiring and wage bargaining is managed by employment agencies. The agencies then supply labor services to establishments at

⁶As noticed by Fujita (2004), the standard search and matching model generates counterfactual shape of the impulse response function of vacancies to labor productivity shocks. The setup proposed by Gertler & Trigari (2009) and Gertler et al. (2008) alleviates this issue.

uniform cost determined on the agencies-establishments side of the labor market, although the wages individual workers receive will differ due to the assumption of staggered real wage contracts.

Each employment agency chooses its desired hiring rate to maximize the value of contracting an extra worker, conditional on the agency-specific wage level:

$$J_t(j) = \max_{x_t(j)} \left\{ \begin{array}{l} \tilde{w}_t - w_t(j) - \frac{\kappa}{2} x_t^2(j) \\ + (\rho + x_t(j)) E_t [\beta \Lambda_{t,t+1} J_{t+1}(j)] \end{array} \right\}$$

The first order condition of the agency can be expressed in the following two forms:

$$\begin{aligned} \kappa x_t(j) &= E_t [\beta \Lambda_{t,t+1} J_{t+1}(j)] \\ \kappa x_t(j) &= E_t \left[\beta \Lambda_{t,t+1} \left[\tilde{w}_{t+1} - w_{t+1}(j) + \frac{\kappa}{2} x_{t+1}^2(j) + \rho \kappa x_{t+1}(j) \right] \right] \end{aligned}$$

and all agencies with the same level of offered wages will choose the same hiring rate.

The workers can be either employed or unemployed, and I denote the values of those states by \mathcal{E} and \mathcal{U} , respectively. The value of being employed by j -th agency is given by:

$$\mathcal{E}_t(j) = w_t(j) + E_t [\beta \Lambda_{t,t+1} [\rho \mathcal{E}_{t+1}(j) + (1 - \rho) \mathcal{U}_{t+1}]]$$

An unemployed worker is a priori uncertain about the wage offer she will receive upon creating a successful match with an agency. By denoting with G the cumulative distribution of wages the expected value of being newly hired is approximated by:

$$\mathcal{E}_t \approx \int \mathcal{E}_t(w_t) dG(w_t)$$

where the approximation is valid up to a first order conditional on wage distribution along the Balanced Growth Path to be degenerate⁷. The value of being unemployed then follows:

$$\mathcal{U}_t = b_t + E_t [\beta \Lambda_{t,t+1} [p_t \mathcal{E}_{t+1} + (1 - p_t) \mathcal{U}_{t+1}]]$$

Accordingly, the surplus of a worker employed by agency j and the average surplus of newly hired workers equal:

$$\begin{aligned} H_t(j) &= \mathcal{E}_t(j) - \mathcal{U}_t \\ H_t &= \mathcal{E}_t - \mathcal{U}_t \end{aligned}$$

And the individual worker's surplus can be rewritten as:

$$H_t(j) = w_t(j) - b_t + E_t [\beta \Lambda_{t,t+1} [\rho H_{t+1}(j) - p_t H_{t+1}]]$$

3.1.6.3 Staggered wage bargaining

The wages are subject to the Calvo-like staggered wage contract friction at the employment agency level, with the average contract duration of $1/(1 - \lambda)$. Therefore, the wage offered by an employment agency is given by:

$$w_t(j) = \begin{cases} w_t(r) & \text{with probability } 1 - \lambda \\ w_{t-1}(j) \cdot Q_t/Q_{t-1} & \text{with probability } \lambda \end{cases}$$

⁷See Gertler & Trigari (2009) for the full argument.

where $w_t(r)$ denotes the wage bargained when employment agencies are allowed to renegotiate. I assume that in the case of being unable to renegotiate wages are indexed with aggregate quality growth. This assumption is necessary for the balanced growth path distribution of wages to collapse to a single point. As a consequence, the average wage will follow the standard Calvo assumption:

$$w_t = \lambda \frac{Q_t}{Q_{t-1}} w_{t-1} + (1 - \lambda) w_t(r) \quad (3.25)$$

An agency that receives a signal to renegotiate in the current period bargains with the marginal worker over the surplus. The bargained contract wage maximizes the following Nash product:

$$w_t(r) = \arg \max H_t(r)^\psi J_t(r)^{1-\psi}$$

where I already impose the Hosios (1990) condition that both sides' bargaining power correspond to matching function elasticities. The first order condition for the Nash bargaining problem is given by:

$$\psi \frac{\partial H_t(r)}{\partial w_t(r)} J_t(r) = (1 - \psi) \frac{\partial J_t(r)}{\partial w_t(r)} H_t(r)$$

While Gertler & Trigari (2009) consider a case where the above formula gives rise to the horizon effect of the agency, the effect disappears under assumption that the wage bargaining and hiring decisions are simultaneous, i.e. internalizing the first order condition of the employment agency⁸. Then the solution of the Nash bargaining problem is of the conventional surplus sharing form:

$$\psi J_t(r) = (1 - \psi) H_t(r)$$

If the wages were renegotiated on the period-by-period basis, then the contract wage would be equal to:

$$w_t^f = \psi \left(\tilde{w}_t + \frac{\kappa}{2} x_t^2 + p_t \kappa x_t \right) + (1 - \psi) b_t \quad (3.26)$$

However, the problem is more involved in the case of staggered contracts. Denote by $W_t(j)$ the expected discounted sum of future wages received over the duration of the relationship with the employment agency:

$$W_t(j) = \Delta_t w_t(j) + (1 - \lambda) E_t \sum_{s=1}^{\infty} (\beta \rho)^s \Lambda_{t,t+s} \Delta_{t+s} w_{t+s}(r)$$

where the first part represents contract that is not renegotiated and the wage is only indexed, while the second part represents future, renegotiated contracts at the same employment agency, and:

$$\Delta_t = E_t \sum_{s=0}^{\infty} (\beta \rho \lambda)^s \Lambda_{t,t+s} \frac{Q_{t+s}}{Q_t} \quad (3.27)$$

⁸In any case, the quantitative impact of the horizon effect is negligible.

The surplus of workers at renegotiating agency can be then rewritten as:

$$\begin{aligned} H_t(r) &= w_t(r) + E_t[\beta\Lambda_{t,t+1}\rho H_{t+1}(r)] - b_t - E_t[\beta\Lambda_{t,t+1}p_t H_{t+1}] \\ &= W_t(r) - E_t \sum_{s=0}^{\infty} (\beta\rho)^s \Lambda_{t,t+s} (b_{t+s} + p_{t+s} H_{t+s+1}) \end{aligned}$$

Similarly, the surplus value of employed worker from the point of view of the employment agency can be rewritten as:

$$\begin{aligned} J_t(r) &= \tilde{w}_t + \frac{\kappa}{2} x_t^2(r) + \rho E_t[\beta\Lambda_{t,t+1} J_{t+1}(r)] - w_t(r) \\ &= E_t \sum_{s=0}^{\infty} (\beta\rho)^s \Lambda_{t,t+s} \left(\tilde{w}_{t+s} + \frac{\kappa}{2} x_{t+s}^2(r) \right) - W_t(r) \end{aligned}$$

By substituting the above expressions in the surplus sharing equation one can obtain:

$$\begin{aligned} W_t(r) &= \psi E_t \sum_{s=0}^{\infty} (\beta\rho)^s \Lambda_{t,t+s} \left(\tilde{w}_{t+s} + \frac{\kappa}{2} x_{t+s}^2(r) \right) \\ &\quad + (1 - \psi) E_t \sum_{s=0}^{\infty} (\beta\rho)^s \Lambda_{t,t+s} (b_{t+s} + p_{t+s} H_{t+s+1}) \end{aligned}$$

or, after simplifying, in the following recursive form:

$$\begin{aligned} \Delta_t w_t(r) &= \psi \left(\tilde{w}_t + \frac{\kappa}{2} x_t^2(r) \right) + (1 - \psi) (b_t + p_t E_t[\beta\Lambda_{t,t+1} H_{t+1}]) \\ &\quad + \rho \lambda E_t[\beta\Lambda_{t,t+1} \Delta_{t+1} w_{t+1}(r)] \end{aligned} \tag{3.28}$$

where the first two terms comprise the target wage w_t^o , which in turn can be expressed in relation to the flexible contract wage:

$$\begin{aligned} w_t^o &= \psi \left(\tilde{w}_t + \frac{\kappa}{2} x_t^2(r) \right) + (1 - \psi) (b_t + p_t E_t[\beta\Lambda_{t,t+1} H_{t+1}]) \\ &= w_t^f + \psi \left(\frac{\kappa}{2} (x_t^2(r) - x_t^2) + p_t \kappa (x_t(r) - x_t) \right) \\ &\quad + (1 - \psi) p_t E_t[\beta\Lambda_{t,t+1} \lambda \Delta_{t+1} (w_{t+1} - w_{t+1}(r))] \end{aligned} \tag{3.29}$$

The above equation emphasizes the presence of spillovers of economy-wide wages on the bargaining wage. Intuitively, more intensive hiring by an agency requires also higher bargained wages, which are also upwardly pressured by the expected future average wage.

Finally, let x_t denote the average hiring rate:

$$x_t = \int_0^1 x_t(j) \frac{n_t(j)}{n_t} dj$$

Then the job creation condition can be used to express x_t as:

$$\begin{aligned} \kappa x_t &= E_t \left[\beta\Lambda_{t,t+1} \left(\tilde{w}_{t+1} - w_{t+1} + \frac{\kappa}{2} x_{t+1}^2 + \rho \kappa x_{t+1} \right) \right] \\ &\quad + E_t \left[\frac{\beta\Lambda_{t,t+1} \int_0^1 \left(\frac{\kappa}{2} x_{t+1}^2(j) + \rho \kappa x_{t+1}(j) - w_{t+1}(j) \right) \frac{n_t(j)}{n_t} dj}{-\left(\frac{\kappa}{2} x_{t+1}^2 + \rho \kappa x_{t+1} - w_{t+1} \right)} \right] \end{aligned} \tag{3.30}$$

Note that along the Balanced Growth Path the deviations of individual employment agencies' decisions from average disappear and as a first order approximation one can take only the first line of the above equation.

3.1.7 Market clearing

Factor markets are assumed to clear at each period:

$$N_t^p = (1 - s) n_t^u \quad \text{and} \quad N_t^s = s n_t^s \quad (3.31)$$

$$K_t = K_t^p + K_t^s \quad (3.32)$$

Supply and demand for skilled inputs are equal:

$$(K_t^s/Q_t)^\alpha (N_t^s)^{1-\alpha} = M_t f + (M_t - M_t^x) \left(\frac{1}{a} \frac{\chi_t}{1 - \chi_t} \right) + \frac{M_t^e}{\chi_t^e} \left(f^e + \frac{1}{a^e} \frac{\chi_t^e}{1 - \chi_t^e} \right) \quad (3.33)$$

where the three sources of demand are: fixed costs of active establishments, R&D activities of incumbents with non-obsolete varieties and fixed costs and R&D activities of prospective entrants.

Finally, the final goods output is spent on consumption, investment and covering hiring costs:

$$Y_t = C_t + K_{t+1} - (1 - dp) K_t + \kappa^u (x_t^u)^2 N_t^p + \kappa^s (x_t^s)^2 N_t^s \quad (3.34)$$

3.2 Data and results

3.2.1 Data, calibration and estimation

The data used in this Chapter come from several major sources. The primary source of data on establishment dynamics comes from the US Bureau of Labor Statistics (BLS) Business Employment Dynamics (BDM) database. Data on GDP, its components and R&D expenditures are provided by the US Bureau of Economic Analyses (BEA), while data on R&D employment come from the National Science Foundation (NSF). Historical establishment employment data are taken from County Business Patterns (CBP). Data on hours and wages are taken from the Nonfarm Business Sector statistics provided by the BLS. Data on unemployment rate and vacancy rate are also taken from the BLS, although for years 1951-2000 the data on vacancies are based on the composite help-wanted index by Barnichon (2010).

The parameters that influence the balanced growth path of the economy are calibrated to reflect the long-run averages in the US data and are summarized in Table 3.1. The values of parameters governing the behavior of the labor markets were taken from previous literature. Differentiated separation rates for unskilled and skilled workers are taken from Cairo & Cajner (2017) and adapted to the quarterly model setup⁹. The adjustment cost parameters were chosen to match the average job finding probability in the US, which Shimer (2005) reports to be equal to 0.45 at monthly frequency and Cairo & Cajner (2017) document that the job finding probabilities differ only slightly among the workers' education groups. As in Shimer (2005) the unemployment benefits are assumed to be equal to 40% of the steady state wage. Following Gertler & Trigari (2009) I set the

⁹Cairo & Cajner (2017) document statistics for workers differentiated by their education level. I treat skilled workers to be analogous to holders of college degree and unskilled to be analogous to high school graduates.

elasticity of matches to unemployment to 0.5 and impose the Hosios (1990) condition that the bargaining power parameters correspond to matching elasticities. Finally, I set the matching efficiency parameter to match the observed average vacancy to unemployment ratio to 0.54, although Shimer (2005) emphasizes that the value of this parameter is virtually irrelevant as beside influencing the average labor market tightness it has no impact on other variables.

Both the capital share of income and quarterly depreciation rate are set to values ubiquitous in the business cycle literature. The discount factor, which in the calibration process depends on the value of elasticity of intertemporal substitution, is chosen so that the average annual net interest rate is equal to 5%. The share of skilled workers is picked to be in the middle of the plausible range of values proposed by Acemoglu et al. (2013) and corresponds to the value used in the previous Chapter and adjusted to account for the presence of unemployment in the model.

Finally, the set of parameters governing the establishment dynamics is calibrated to match specific moments reported in Table 3.2. As I have 6 moments to match with 8 free parameters, I impose a constraint that the R&D efficiency parameter and fixed cost are equal for both incumbents and entrants.

Table 3.1: Calibrated parameters affecting the steady state

Parameter	Description	Value	Justification
ρ^u	Unskilled retention rate	0.9725 ³	Cairo & Cajner (2017)
ρ^s	Skilled retention rate	0.99 ³	Cairo & Cajner (2017)
κ^u	Unskilled hiring cost	2	Unskilled job finding probability
κ^s	Skilled hiring cost	15.8	Skilled job finding probability
b^u	Unskilled unemp. benefit	0.14	40% of steady state unskilled wage
b^s	Skilled unemp. benefit	0.41	40% of steady state skilled wage
ψ	Elasticity of matches	0.5	Gertler & Trigari (2009)
σ_m	Matching efficiency	1.7	Average tightness = 0.54
α	Capital share of income	0.3	Standard
dp	Capital depreciation rate	0.025	Standard
β	Discount factor	0.9996	Annual net interest rate of 5%
s	Share of skilled workers	0.1039	Chapter 2
ι	Innovative step size	1.016	Annual pc. GDP growth
δ^{exo}	Exog. exit shock prob.	0.0174	Exit rate
a, a^e	R&D efficiency	7.96	Expansions = contractions
f, f^e	Fixed cost	0.94	Share of R&D in GDP
θ	Inverse of IES	2.3	Share of investment in GDP
σ	Elasticity of substitution	4.9	Share of R&D employment

Table 3.2: Long-run moments: comparison of model and data

Description	Model	Data	Source
Annual pc. GDP growth	2.07%	2.08%	BEA, 1948q1-2016q2
Exit rate ^a	3.07%	3.07%	BDM, 1992q3-2016q2
Relative share of expanding estabs.	1.01	1.01	BDM, 1992q3-2016q2
Share of R&D in GDP	2.21%	2.23%	BEA, 1948q1-2016q2
Share of investment in GDP	16.91%	17.17%	BEA, 1948q1-2016q2
Share of R&D employment	1.25%	0.98%	NSF & CBP, 1964-2008

^aCalculated from the data as the average between death and birth rates.

To obtain the values of parameters that do not affect the steady state but govern the cyclical behavior of the model, I employ the estimation procedure. The prior distributions were chosen to be relatively uninformative, and in particular the prior distribution for the renegotiation frequency parameter was set to uniform on the unit interval. Table A.1 in the Appendix contains full information on the priors used.

The observable variable used in the estimation is the quarterly growth rate of Real Gross Domestic Product divided by the Labor Force, observed in periods 1948q2-2017q2. An advantage of the model with explicitly modeled long-run growth is that there is no need to detrend the data and valuable information is retained. The model was estimated using standard Bayesian procedures with help of Dynare 4.5 and results were generated using two random walk Metropolis-Hastings chains with 200,000 draws each with an acceptance ratio of around 0.23. Figures A.1, A.2 and A.3 in the Appendix demonstrate that the estimation procedure has been able to converge successfully.

Table 3.3 presents the estimation results. The data were clearly informative about the estimated parameters, as the posterior and prior means differ significantly and the highest posterior density (HPD) intervals are relatively tight. This observation can be also confirmed by comparing the plots of prior and posterior densities displayed in Figure 3.1.

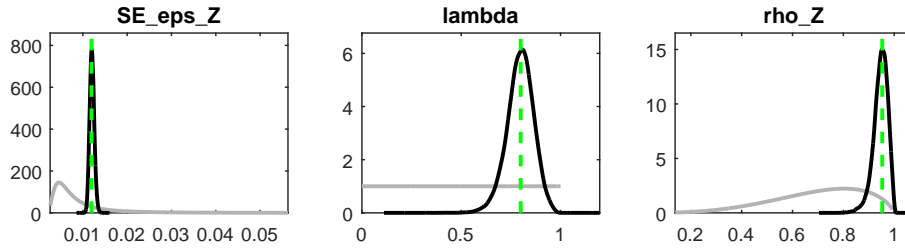
The most interesting parameter here is λ that determines contract renegotiation probability, and its value implies that wage contracts last on average for 5 quarters. This value is slightly higher than assumed by Gertler & Trigari (2009) in their calibrated model, where they consider average duration of 9 and 12 months, and also higher than estimated by Gertler et al. (2008) where contracts last for 3.5 quarters¹⁰. However, assuming this value of the parameter yields excellent performance in case of labor market variables, which were not observed directly during the estimation procedure (see Table 3.5).

¹⁰Note however that Gertler et al. (2008) impose a relatively tight prior on this parameter.

Table 3.3: Prior and posterior means of parameters affecting cyclical behavior

Parameter	Description	Prior mean	Post. mean	90% HPD interval
λ	Calvo parameter (wages)	0.5	0.796	[0.691, 0.909]
ρ_Z	Autocorr. of TFP process	0.7	0.946	[0.905, 0.990]
σ_Z	Std. dev. of TFP shock	0.01	0.012	[0.011, 0.013]

Figure 3.1: Prior and posterior distributions



3.2.2 Model performance and impulse response functions

Table 3.4 presents the comparison of the Hodrick-Prescott filtered moments between the model and data. Data for the variables presented in the upper and middle parts of the table are based on the 1951q1-2016q4 sample. Output is based on the Gross Domestic Product by BEA, consumption on the sum of Personal Consumption Expenditures on Nondurable Goods and Services, investment on the sum of Personal Consumption Expenditures on Durable Goods and Fixed Private Investment, and R&D expenditures on Gross Domestic Product: Research and Development. Wages are based on Nonfarm Business Sector: Compensation Per Hour by BLS, and hours on Hours of All Persons. Unemployment rate is taken from the BLS, and vacancy rate is taken from JOLTS by BLS and spliced with composite help-wanted index by Barnichon (2010). Data for variables presented in the lower part of the table are based on the 1992q3-2016q4 sample, covering 99 periods, and come from the BDM. All variables trending with population size were divided by the Civilian Labor Force by BLS, and variables in nominal terms were deflated by the Gross Domestic Product: Implicit Price Deflator by BEA.

The upper section of the table is concerned with output and its components, as well as R&D expenditures. The model fits the data very well for output and its components, and only fails to account for much weaker correlation of R&D expenditures with output.

The middle section of the table focuses on variables pertaining to the operations of the labor market. The model wages are stronger correlated with output and have higher autocorrelation than in the data, and model hours are not as volatile as in the data. However, the model is very successful in matching the cyclical behavior of unemployment, vacancies and tightness, achieving nearly perfect fit. Additionally, Table 3.5 presents

correlations between key labor market variables and confirms that the model is able to replicate the Beveridge curve comovements.

The final section presents the moments related to the establishment dynamics. Although the fit is a bit worse than in the case of previously discussed variables, most of the model moments remain close to their data counterparts, with the exception that the model predicts much smaller volatility of establishment dynamics. The model also predicts that the establishment mass is slightly negatively correlated with output, even though the correlation of net entry with output is almost exactly the same as in the data. A brief look at the impulse response functions in Figure 3.2 reveals that this result is most likely driven by a small and short-lived decrease in the mass of establishments immediately after the shock hits, and for the subsequent periods the mass of active establishments moves in tandem with output.

To sum up, although the model is not able to match the data perfectly, the fit is more than satisfactory and provides a solid foundation for further analysis.

Table 3.4: Business cycle moments: comparison of model and data

Variable	Standard deviation		Correlation with Y		Autocorrelation	
	Data	Model	Data	Model	Data	Model
Output	1.58	1.58	1.00	1.00	0.82	0.83
Consumption	0.87	0.80	0.78	0.98	0.82	0.75
Investment	4.54	5.55	0.76	0.99	0.87	0.87
R&D	2.36	2.07	0.32	0.94	0.89	0.92
Wages	0.95	0.82	0.10	0.54	0.68	0.96
Hours	1.36	0.65	0.86	0.93	0.89	0.91
Unemployment	12.76	10.80	-0.77	-0.80	0.89	0.91
Vacancies	13.78	12.80	0.83	0.94	0.91	0.88
Tightness	26.00	22.57	0.82	0.92	0.91	0.92
Establishments	0.62	0.22	0.71	-0.14	0.87	0.92
Expansions	2.84	0.47	0.82	0.65	0.75	0.95
Contractions	2.38	0.42	-0.11	-0.89	0.69	0.91
Net Entry	0.31	0.09	0.31	0.35	0.24	0.51

Table 3.5: Correlations between labor market variables

Correlation	Data	Model
Unemployment, Vacancies	-0.92	-0.82
Tightness, Unemployment	-0.98	-0.95
Tightness, Vacancies	0.98	0.96

Figure 3.2 displays the impulse response functions to a 1% productivity shock. An increase in productivity raises output directly, but also induces higher investment which

raises the stock of physical capital, and it also makes hiring more attractive, which reduces unemployment and increases hours worked in the economy. The response of output to the shock is highly persistent, both due to labor market frictions and the endogenous quality component which permanently shifts output upwards. Expenditures on R&D are also procyclical and persistent.

Due to staggered wage contracts average wages respond on impact quite modestly as a large fraction of labor agencies are unable to renegotiate the wages. The impulse response of wages displays a hump-shaped pattern, reaching its peak around 3 years after the shock hits. Increased productivity of labor induces the employment agencies to post vacancies, increasing labor market tightness, which subsequently increases employment and thus hours worked.

Figure 3.2: Impulse response functions to 1% productivity shock (%)

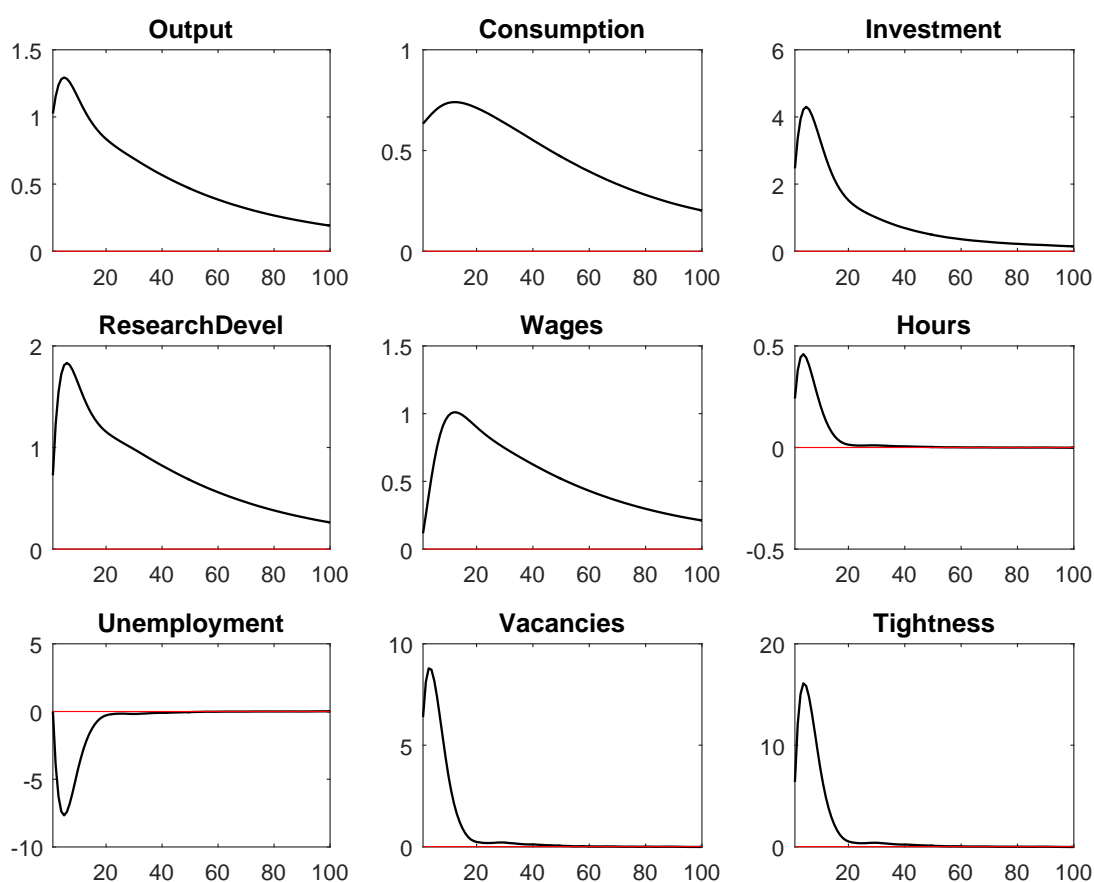
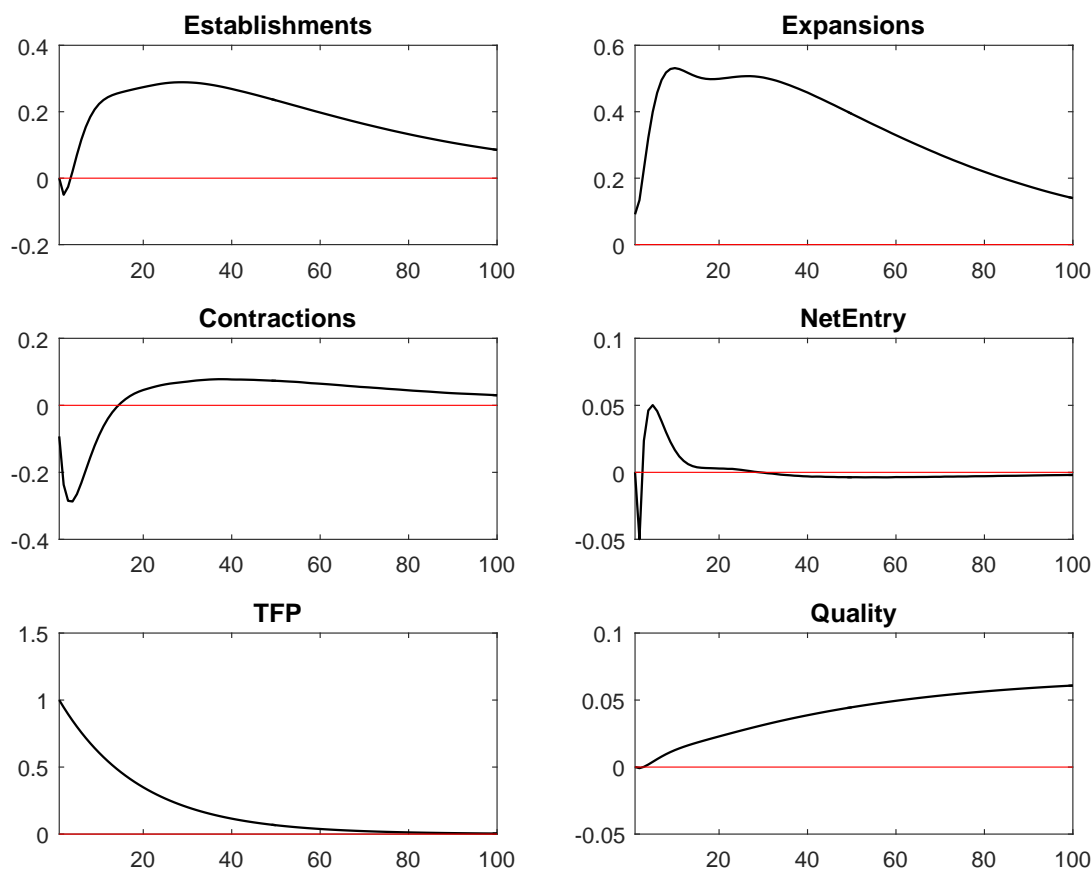


Figure 3.3 displays the impulse response functions of establishment dynamics. Following the productivity shock incumbents increase their R&D intensity, and the mass of expanding establishments increases while the mass of contracting establishments decreases. The increased demand from incumbents for scarce skilled labor results in a brief reduction in net entry rates, which translates to a small decrease in the mass of establishments. As the mass of employed skilled workers increases due to elevated hiring, net entry becomes positive and the mass of establishments increases substantially. Both elevated intensity of R&D by the incumbents and higher entry lead to an increase in the rate of growth of the aggregate quality index. For the first 5 years after the shock the increase in quality is fueled both by higher employment of skilled workers and bigger stock of physical capital, afterwards only more abundant physical capital maintains faster growth in quality level. The level of quality flattens out gradually and stabilizes at a level around 7% higher than it would be absent the shock.

As a robustness check, Figure A.4 in the Appendix presents the Bayesian impulse response functions taking into account parameter uncertainty. All of the results remain unchanged.

Figure 3.3: Impulse response functions to 1% productivity shock, continued (%)



3.3 Policy implications

The previous section documents the hysteresis effect of temporary shocks on the level of the balanced growth path of the economy. This implies that business cycle fluctuations bear additional welfare costs which are unaccounted for in the models where growth results from exogenous processes.

To quantify the welfare comparisons across different states of the world, I employ the consumption equivalent transformation. The consumption equivalent is equal to the lifetime percentage change in the path of households' consumption that make them indifferent across "living" in two distinct states of the world. The consumption equivalent-adjusted lifetime utility is given by

$$W_0(eq) = E_0 \sum_{t=0}^{\infty} \beta^t \frac{((1+eq) c_t)^{1-\theta}}{1-\theta} = (1+eq)^{1-\theta} E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\theta}}{1-\theta}$$

The consumption equivalent across two different worlds can be then computed as follows:

$$eq_{a,b} = \left(\frac{U_0^b}{U_0^a} \right)^{\frac{1}{1-\theta}} - 1$$

where U_0^a and U_0^b denote expected lifetime utilities in worlds a and b , respectively. Then $eq_{a,b}$ has the interpretation of which proportion of consumption the agent living in world a would be willing to forfeit in order to "move" to world b .

Table 3.6 presents the comparison of expected lifetime utilities in three distinct worlds: non-stochastic, where the economy is not subject to shocks and always remains on its balanced growth path, and two stochastic worlds. In the first of them growth is fully exogenous and the quality index does not react in response to stochastic shocks. The second stochastic world represents the model economy.

The welfare effect of business cycles in the stochastic world with exogenous growth is very small in magnitude and actually indicates welfare gain. The reason for that is that an economy with physical capital has on average higher stock of capital when subject to stochastic shocks, as agents engage in precautionary saving to better smooth their consumption. This in turn implies that the average level of output, and also consumption, are also higher. As the welfare costs of volatility around an invariant trend are minuscule, the level effect dominates. This is a standard result in the business cycle literature.

On the other hand, the welfare costs of business cycles under endogenous growth are substantial. Since the transitory shocks leave lasting impacts on the level of BGP, it increases dramatically the uncertainty about future consumption paths. As a result, agents would require a compensation of 5.8% of their consumption in order to be indifferent between living in the stochastic and non-stochastic worlds.

Table 3.6: Welfare cost of business cycles

State of the world	Welfare	Consumption equivalent
Non-stochastic (BGP)	-177.55	—
Stochastic with exogenous growth	-177.46	-0.04%
Stochastic with endogenous growth	-191.04	5.79%

Due to the presence of significant welfare costs of business cycles and the potential ability to affect the growth rate of the economy, ample space for policy intervention arises. I analyze the effects of employing two types of subsidy schemes: static and countercyclical, financed through a lump-sum tax/transfer scheme.

In the static case the subsidy acts as if a certain parameter was lowered or raised by 10%. Accordingly, a subsidy to operation cost acts as if the costs themselves were 10% lower, and subsidies to R&D act as if the research efficiency was 10% higher. Table 3.7 presents the results of subsidizing operation cost of incumbents and prospective entrants, their R&D expenditures, and the costs of hiring. Lastly, although it cannot be treated as a subsidy, I analyze the effects of increasing the labor contract renegotiation probability by 10%. In the last column I report the consumption equivalent multiplied by negative one, so that a positive value of the statistic indicates welfare gain.

The results indicate that subsidizing both operating cost and R&D expenditures of incumbent establishments is strongly welfare improving. This result may be surprising in the perspective of existing endogenous growth literature that almost unanimously generates result that subsidizing operating costs of incumbents is welfare deteriorating, as in e.g. Acemoglu et al. (2013). The reason I obtain the opposite results stems from the fact that my model features a frictional labor market. As can be seen in Table 3.7, subsidizing incumbents' operational cost leads to much lower rate of unemployment, as an effect of decreased churning in the labor market and higher establishment mass. This results in a higher level of aggregate output, as both the employment and love-for-variety effects move in the same direction. The static level gain dominates the effects that stem from lower rate of growth of the economy.

The remaining results have a very intuitive interpretation. In general, households prefer to live in worlds with *ceteris paribus* higher growth rates, lower volatility and lower unemployment rates. The subsidy to entrants' operating cost helps in lowering the unemployment rate and generates welfare gain even though the growth rate is slightly lower and the economy is slightly more volatile. As already discussed, subsidies to incumbents' R&D expenditures give rise to significant welfare gains, as despite slightly elevated unemployment rates the rate of growth of economy is much higher and it is less volatile. The small positive welfare effect from subsidizing entrants' R&D stems from lower unemployment rate. Decreasing the hiring costs in the labor market, both for the unskilled and skilled workers, generates welfare improvement, mostly stemming from decreased unemployment rates. What is important, subsidizing the hiring in the unskilled labor market where the majority of workers operate, yields also smaller volatility of the economy. Finally, increasing contract renegotiation frequency is welfare improving, although the consumption equivalent is rather small.

Table 3.7: Effects of static subsidies

	γ^{BGP}	γ	ΔQ_{20}	ΔQ_{100}	U^{BGP}	U	u^{BGP}	u	$-eq$
Baseline	2.07	2.09	2.23	5.75	-177.55	-191.04	5.65	5.70	—
f	2.03	2.05	2.35	5.98	-167.63	-180.89	5.18	5.24	4.11%
f^e	2.06	2.08	2.25	5.78	-177.16	-190.63	5.63	5.68	0.16%
a	2.12	2.14	2.18	5.57	-174.51	-186.74	5.68	5.73	1.74%
a^e	2.07	2.08	2.24	5.76	-177.41	-190.89	5.64	5.69	0.06%
κ^u	2.07	2.09	2.20	5.67	-175.57	-189.22	5.13	5.19	0.73%
κ^s	2.07	2.09	2.29	5.83	-177.29	-190.90	5.62	5.68	0.06%
λ	2.07	2.08	1.80	5.15	-177.55	-190.94	5.65	5.64	0.04%

Table 3.8 reports the welfare effects of applying countercyclical subsidies. The subsidy scheme works as follows: if output is 1% below trend, the subsidy increases by 0.5%. As such, it is actually a tax in the boom periods. The results fall in line with those obtained in the previous Chapter. Countercyclical subsidies to operating costs of both incumbents and entrants are welfare enhancing. On the other hand, subsidizing incumbents' R&D expenditures takes away precious resources from entrants when they need them most, and generates a significant welfare loss. Finally, countercyclical hiring subsidies generate a negligible positive welfare effect.

Table 3.8: Effects of countercyclical subsidies

	ΔQ_{20}	ΔQ_{100}	U	u	$-eq$
Baseline	2.23	5.75	-191.04	5.698	—
f	2.89	7.50	-190.33	5.687	0.28%
f^e	2.27	5.86	-190.99	5.698	0.02%
a	0.96	2.64	-195.16	5.700	-1.66%
a^e	2.25	5.79	-191.00	5.698	0.02%
κ^u	2.22	5.73	-191.07	5.695	0.01%
κ^s	2.23	5.75	-191.03	5.698	0.00%

To sum up, the most welfare improving subsidies are static subsidies to incumbents' operating cost and R&D expenditures, and countercyclical subsidies to incumbents' operating cost. This provides justification for policies aiming to decrease firm exit during recessions.

3.4 Conclusions

In this Chapter I have extended the model by including physical capital accumulation and labor market frictions modeled as in Gertler et al. (2008) and Gertler & Trigari (2009). The model is able to generate volatile and procyclical R&D expenditure patterns and is

consistent with the business cycle dynamics of GDP and its components, labor market variables, as well as establishment dynamics.

The model makes predictions on the strength of the impact of business cycle fluctuations on the endogenous growth rates of the economy. Similarly to Chapter 2, the results suggest that the mechanism governing innovation dynamics generates hysteresis effects of temporary shocks on the BGP level, translating around 7% of the strength of a shock to the level shift of the BGP, impacting significantly the assessment of welfare costs of business cycles.

As in Chapter 2 I find that the welfare effects of business cycles are nontrivial and of two orders of magnitude higher than in the models with exogenous growth. Considerable welfare effects and the potential to influence endogenous growth rates creates ample scope for policy intervention. I examine the welfare effects of both static and countercyclical subsidy schemes.

In line with the extant endogenous growth literature, I find that static subsidies to R&D, as well as to the entrants, are welfare improving. In opposition to the previous results in the literature, I find that subsidizing incumbent firms generates large and positive welfare effects, as the static gains of bigger number of firms active in the market, leading to lower unemployment and love-for-variety effects, dwarf dynamic losses of lowered entry rates. I also confirm that decreasing frictions in labor markets is welfare improving.

In the case of countercyclical subsidies I find that subsidizing incumbents' R&D expenditures is welfare deteriorating, while subsidizing their operating costs is welfare enhancing. This gives further support for policies designed to subsidize existing firms during recessions.

Chapter 4

Long shadows of financial shocks: an endogenous growth perspective

The experience of the Great Recession and its aftermath has compelled many macroeconomists to examine links between the financial sector and real activity. This Chapter extends the model presented in the two previous Chapters by incorporating a form of financial frictions by assuming the working capital requirement as in Christiano et al. (2003, 2010) and having a reduced form of financial shocks in the form of the spread between the deposit and lending interest rates. There already exists literature that recognizes the impact of financial disturbances on macroeconomic variables¹ and firm dynamics, especially in the context of Great Recession. Aghion et al. (2010) study the effects of credit friction on the cyclical composition of investment and find that credit constraints can both decrease growth and increase the economy's volatility. Severo & Estevao (2010) use industry-level panel data from Canada and US to show that increases in the cost of funds for firms have negative effects on TFP growth. Queralto (2011) documents that financial crises in emerging countries involve large and persistent losses in labor productivity. Fernandez-Corugedo et al. (2011) build a DSGE model with multiple components of the working capital channel and find that even under flexible prices a disruption to the supply of credit has large and persistent effects on the real economy. Ates & Saffie (2014) build an small open economy entry-driven endogenous growth model to analyze the effects of a sudden stop using Chilean plant-level data. They find that although during financial shortage entrants are usually better, but they are fewer, generating permanent loss of output and significant welfare costs. Siemer (2014) finds that tight financial constraints during the Great Recession were responsible for both low employment growth and firm entry rates. Hall (2015) documents the effects of the Great Recession on the US economy and finds multiple negative phenomena, such as depressed investment, reallocation, and job finding rates. Moreover, he points out that of the 13% drop in 2013 level of output relative to the pre-recession trend, 3.5% was due to the shortfall in total factor productivity. Christiano et al. (2015) build a medium-scale DSGE model to quantify the importance of several shocks during the Great Recession and find that financial frictions were driving a significant part of the macroeconomic variables' behavior. This Chapter builds on this literature by considering the effects of financial frictions within a model that

¹Brunnermeier et al. (2012) provide an exhaustive review of the macroeconomic effects of financial frictions.

simultaneously accounts for rich firm dynamics, entry and exit decisions, and endogenous growth.

The Chapter is organized as follows. The next section describes the model, focusing on presenting the problem of incumbents and potential entrants, and describing the labor market and financial frictions. The second section provides a description of the data sources, parameter values and the estimation procedure, and discusses the cyclical properties of the model economy against the data. The third section applies the model to quantify the relative importance of the shocks during the Great Recession. It also analyzes in depth the welfare properties of the model economy and presents the macroeconomic and welfare effects of applying several subsidy schemes, both static and counterfactual. The last section concludes.

4.1 Model

Compared to the previous Chapter, the model introduces two key changes. First, as I am interested in the effect of financial shocks on the model macroeconomy, I introduce the wedge between the interest rate depositors (households) receive and the interest rate borrowers (establishments) pay. This allows me to capture the effects of a temporarily increased risk premium. Second, in line with Christiano et al. (2003, 2010), I incorporate the working capital channel which was found to be an important amplifier of financial shocks to the real economy.

4.1.1 Households

The mass of representative households is normalized to unity. Each of the households is composed of a large family of workers who differ with respect to their employment status and skill level. Nevertheless, due to within-family sharing, all individuals enjoy identical levels of consumption. The representative household maximizes the lifetime expected utility:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\theta}}{1-\theta} \quad (4.1)$$

where c_t denotes per capita consumption, β represents the discount factor and θ is the inverse of the elasticity of intertemporal substitution.

Following Acemoglu et al. (2013), I assume that workers belong to either of the two skill groups: unskilled workers of aggregate mass $(1 - s)$ supply labor to the production sector, while skilled of aggregate mass s are hired as managers or to perform research and development activities. Furthermore, within a time period an individual worker may be employed and receive wage income, or unemployed and receive unemployment benefits. Here I abstract from the possibility that an individual is inactive in the labor market². The budget constraint of a representative household is given by:

$$c_t + d_{t+1} = (1 - s) [w_t^u n_t^u + b_t^u (1 - n_t^u)] + s [w_t^s n_t^s + b_t^s (1 - n_t^s)] + (1 + r_t^d) d_t + t_t$$

²The setup of the labor markets is identical to the one considered in the previous Chapter, and for the sake of brevity I do not repeat the derivation in this Chapter..

where d_{t+1} is the end-of-period t stock of deposits supplied to the financial sector which yields interest at deposit rate r_t^d , w_t^u and w_t^s represent wage income of employed unskilled and skilled workers, respectively, while n_t^u and n_t^s are the employment rates, and b_t^u and b_t^s denote unemployment benefits. Finally, t_t represents the sum of all dividend payments and lump-sum transfers net of taxes received by the households.

The intertemporal optimization by households yields the Euler equation:

$$c_t^{-\theta} = \beta E_t \left[(1 + r_{t+1}^d) c_{t+1}^{-\theta} \right] \quad (4.2)$$

and it is also convenient to define the stochastic discount factor of the households:

$$\Lambda_{t,t+1} = E_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\theta} \right] \quad (4.3)$$

By assuming that all firms are ultimately owned by households, I impose the condition that their managers discount future profit streams according to the households' valuation depending on the expected relative marginal utilities of consumption.

4.1.2 Final goods producers

The perfectly competitive final goods producers purchase differentiated intermediate goods varieties and transform them into final goods via the CES aggregator:

$$Y_t = \left[\int_0^{M_t} y_t(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}$$

where Y_t is the aggregate final goods output, M_t is the mass of active intermediate goods producing establishments, $y_t(i)$ is the quantity demanded from the i -th producer, and σ is the elasticity of substitution between the varieties.

Solving the profit maximization problem of the final goods producers yields the following Hicksian demand function for the i -th variety:

$$y_t(i) = Y_t p_t(i)^{-\sigma}$$

where $p_t(i)$ denotes the i -th variety's price relative to the aggregate price index, which can be constructed as follows:

$$P_t = \left[\int_0^{M_t} P_t(i)^{1-\sigma} di \right]^{\frac{1}{1-\sigma}}$$

and where $P_t(i)$ is the nominal price for the unit of the i -th intermediate good.

4.1.3 Intermediate goods producers

The monopolistically competitive intermediate goods first have to bear a fixed cost of operation f_t , which represents the costs of management and other non-production activities. Subsequently they can produce by employing capital and unskilled labor services according to the following Cobb-Douglas function:

$$y_t(i) = Z_t k_t^p(i)^\alpha [q_t(i) n_t^p(i)]^{1-\alpha}$$

where Z_t denotes the stochastic aggregate TFP parameter, $k_t^p(i)$ and $n_t^p(i)$ are the employed capital and unskilled labor, respectively, and $q_t(i)$ represents the idiosyncratic quality level of the i -th variety at time period t . Parameter α describes the elasticity of intermediate goods output with respect to capital.

The intermediate goods producers choose such combination of capital and labor that minimizes their costs. As in Christiano et al. (2003, 2010) I assume that each producer has to finance a constant fraction ζ of both the capital rental cost and wage bill in advance of production by borrowing necessary funds at the lending rate r_t^{l3} . The solution of the cost minimization problem results in the following expression for the marginal cost:

$$mc_t^p(i) = \frac{(1 + \zeta r_t^l)}{Z_t} \left(\frac{r_t^k}{\alpha} \right)^\alpha \left(\frac{\tilde{w}_t^u / q_t(i)}{1 - \alpha} \right)^{1-\alpha}$$

where r_t^k is the real rental rate on capital and \tilde{w}_t^u denotes the unskilled wage paid to the employment agency. Note that the marginal costs differ across intermediate goods producers due to their differentiated quality levels.

Given that the producers can freely change their prices on the period-by-period basis, the optimal pricing strategy is achieved by applying a constant markup over the marginal cost:

$$p_t(i) = \frac{\sigma}{\sigma - 1} mc_t(i)$$

As in Melitz (2003), I construct the aggregate quality index Q_t which will summarize the aggregate situation in the intermediate goods sector as if it was populated by mass M_t of producers each with the same quality level. The index is constructed by applying the following formula:

$$Q_t = \left[\int_0^\infty q^{(1-\alpha)(\sigma-1)} \mu_t(q) dq \right]^{\frac{1}{(1-\alpha)(\sigma-1)}}$$

where $\mu_t(q)$ denotes the period t distribution of the idiosyncratic quality levels.

It is very useful to describe the situation of an individual producer by comparing its quality level to the aggregate quality index and expressing it in relative terms:

$$\phi_t(i) = \left(\frac{q_t(i)}{Q_t} \right)^{(1-\alpha)(\sigma-1)}$$

It can then be shown that the operating profit of an intermediate goods producer can be expressed as:

$$\pi_t^o(i) = \frac{Y_t}{\sigma M_t} \phi_t(i) - f_t$$

Moreover, the aggregate final goods output is given as follows:

$$Y_t = M_t^{\frac{1}{\sigma-1}} Z_t (K_t^p)^\alpha (Q_t N_t^p)^{1-\alpha} \quad (4.4)$$

³In principle Christiano et al. (2003, 2010) allow for the degree of pre-financing to differ between the payments to capital and labor, but eventually they also assume that they are equal. I impose this assumption from the start and save on notation.

where K_t^p and N_t^p denote, respectively, the aggregate employment of capital and unskilled labor in the production sector while the presence of the active producers mass in the expression results from the love-for-variety phenomenon. Note that in the long run the only source of economic growth is the continued increase in aggregate quality level over time, and both capital stock and output will grow at the corresponding rate.

4.1.4 Incumbents

In the previous subsection I have discussed the static problem of the intermediate goods producer, where the quality level was given. This subsection describes the problem in the dynamic setting, where the incumbent producers can engage in research and development activities to have a chance at increasing their varieties' quality. The relative quality level of the i -th variety in period $t + 1$ is decided by the following lottery:

$$\phi_{t+1}(i) = \begin{cases} \iota \phi_t(i) / \eta_t & \text{with probability } \chi_t(i) \\ \phi_t(i) / \eta_t & \text{with probability } 1 - \chi_t(i) \end{cases}$$

where ι denotes the size of an innovative step and η_t is the transformed rate of growth of the aggregate quality index, which individual producers take as given:

$$\eta_t = \left(\frac{Q_{t+1}}{Q_t} \right)^{(1-\alpha)(\sigma-1)}$$

The innovative success probability $\chi_t(i)$ is chosen endogenously by each producer and is a function of engaged R&D resources. The particular form of the success probability function is based on Pakes & McGuire (1994) and Ericson & Pakes (1995):

$$\chi_t(i) = \frac{ax_t(i)}{1 + ax_t(i)}$$

where a is a parameter that describes the efficiency of R&D input $x_t(i)$. The R&D process requires hiring both capital and skilled labor:

$$x_t(i) = \frac{k_t^x(i)^\alpha [Q_t n_t^x(i)]^{1-\alpha}}{Q_t \phi_t(i)}$$

where $k_t^x(i)$ and $n_t^x(i)$ denote the employed of capital and skilled labor.

The presence of aggregate and relative quality levels in the expression lends itself to a very intuitive interpretation. Aggregate quality level in the numerator multiplies with R&D laborers as they have access to a pool of common knowledge. However, over time it is harder to come up with new ideas unless more resources are committed to R&D activities, which is captured by aggregate quality level in the denominator. Finally, the presence of relative quality level in the denominator represents the catch-up and headwind effects, depending on establishments' position in the quality distribution.

The solution of the cost minimization problem results in the following expression for the marginal cost of R&D activities:

$$mc_t^x(i) = (1 + \zeta r_t^l) Q_t^\alpha \left(\frac{r_t^k}{\alpha} \right)^\alpha \left(\frac{\tilde{w}_t^s}{1 - \alpha} \right)^{1-\alpha} \phi_t(i) \equiv (1 + \zeta r_t^l) \bar{m}c_t^x \phi_t(i)$$

where \tilde{w}_t^s denotes the skilled wage paid to the employment agency and $\bar{m}c_t^x$ denotes the skilled marginal cost component common to all establishments.

To simplify the setup of the intermediate goods producer, I assume that managerial activities require identical combination of capital and skilled labor as R&D activities. The fixed cost of operation can then be rewritten as follows:

$$f_t = (1 + \zeta r_t^l) \bar{m}c_t^x f$$

Furthermore, to simplify notation I define the cost of skilled input relative to the current aggregate final goods output:

$$\omega_t = \bar{m}c_t^x / Y_t \quad (4.5)$$

The real profit of an incumbent can then be expressed as the following function, affine in terms of the relative quality level:

$$\pi_t(i) = Y_t \left[\left(\frac{1}{\sigma M_t} - (1 + \zeta r_t^l) \frac{\omega_t}{a} \frac{\chi_t(i)}{1 - \chi_t(i)} \right) \phi_t(i) - (1 + \zeta r_t^l) \omega_t f \right]$$

Since all producers with the same relative quality levels will behave identically, from now on I drop the subscript i . The value of a producer with relative quality level ϕ_t can be expressed as follows:

$$V_t(\phi_t) = \max_{\chi_t \in [0,1]} \{ \pi_t(\phi_t, \chi_t) + \max \{0, E_t [\beta \Lambda_{t,t+1} (1 - \delta_t) V_{t+1}(\phi_{t+1} | \phi_t, \chi_t)] \} \}$$

where δ_t is an endogenous probability that a producer will receive a death shock and the $\max \{0, \cdot\}$ operator allows the producers with low enough relative quality levels to voluntarily exit when the expected continuation value turns negative.

As the aggregate quality level trends upwards over time, causing other variables to trend as well, the above expression does not lend itself well to casting in the recursive form. I employ the following stationarization:

$$v_t(\phi_t) = \frac{V_t(\phi_t)}{Y_t}$$

where $v_t(\phi_t)$ denotes now the value of a producer relative to the current aggregate final goods output. The normalized value function is now stationary and can be stated as:

$$v_t(\phi_t) = \max_{\chi_t \in [0,1]} \left\{ \frac{\pi_t(\phi_t, \chi_t)}{Y_t} + \max \left\{ 0, E_t \left[\beta \Lambda_{t,t+1} (1 - \delta_t) \left(\frac{Y_{t+1}}{Y_t} \right) v_{t+1}(\phi_{t+1} | \phi_t, \chi_t) \right] \right\} \right\}$$

As the model will be solved via the perturbation methods, I need to split the population of producers into two groups: those for which the $\max \{0, \cdot\}$ operator is not binding, and those that exit.

As far as the first group is concerned, for large enough ϕ_t the probability that the particular producer shall exit in the foreseeable future is negligible and the operator can be safely omitted. Then it is trivial to show that since the value function is a weighted

sum of current and future profit flows, all of which are affine in ϕ_t , then the value function is also affine in ϕ_t and the following functional form can be imposed:

$$A_t + B_t \phi_t = \max_{\chi_t \in [0,1]} \left\{ \left(\frac{1}{\sigma M_t} - \left(1 + \zeta r_t^l\right) \frac{\omega_t}{a} \frac{\chi_t}{1 - \chi_t} \right) \phi_t - \left(1 + \zeta r_t^l\right) \omega_t f \right. \\ \left. + E_t \left[\beta \Lambda_{t,t+1} (1 - \delta_t) \left(\frac{Y_{t+1}}{Y_t} \right) \left(A_{t+1} + B_{t+1} \frac{\chi_t (\iota - 1) + 1}{\eta_t} \phi_t \right) \right] \right\} \quad (4.6)$$

where A_t and B_t are state-dependent coefficients that fluctuate over the business cycle.

The first order and envelope conditions of those producers can be then stated as follows:

$$0 = - \left(1 + \zeta r_t^l\right) \frac{\omega_t}{a} \frac{1}{(1 - \chi_t)^2} + E_t \left[\beta \Lambda_{t,t+1} (1 - \delta_t) \left(\frac{Y_{t+1}}{Y_t} \right) \left(B_{t+1} \frac{(\iota - 1)}{\eta_t} \right) \right] \quad (4.7)$$

$$B_t = \left(\frac{1}{\sigma M_t} - \left(1 + \zeta r_t^l\right) \frac{\omega_t}{a} \frac{\chi_t}{1 - \chi_t} \right) + E_t \left[\beta \Lambda_{t,t+1} (1 - \delta_t) \left(\frac{Y_{t+1}}{Y_t} \right) B_{t+1} \frac{\chi_t (\iota - 1) + 1}{\eta_t} \right] \quad (4.8)$$

As the relative quality level drops out of the above optimality conditions, one can conclude that all producers with high enough relative quality level will choose the same success probability, and their size and growth rate will be uncorrelated, as postulated by the Gibrat's law.

The second group consists of the producers with negative continuation value and they will exit at the end of the current period and optimally choose not to engage in R&D activities at all. Their value function is then also affine in relative quality level and is equal to:

$$v_t(\phi_t) = Y_t \left[\frac{1}{\sigma M_t} \phi_t(i) - \left(1 + \zeta r_t^l\right) \omega_t f \right] \quad (4.9)$$

At this stage the above division does not account for producers with intermediate quality levels. I then impose that all continuing producers have to choose the same success probability as their higher quality competitors. This results in the extension of Equations 4.6 and 4.9 until they intersect at the relative quality level at which a producer is indifferent between exiting and continuing conditional on choosing the common R&D intensity. This level is given implicitly by the following condition:

$$\left(1 + \zeta r_t^l\right) \frac{\omega_t}{a} \frac{\chi_t}{1 - \chi_t} \phi_t^* = E_t \left[\beta \Lambda_{t,t+1} (1 - \delta_t) \left(\frac{Y_{t+1}}{Y_t} \right) \left(A_{t+1} + B_{t+1} \frac{\chi_t (\iota - 1) + 1}{\eta_t} \phi_t^* \right) \right] \quad (4.10)$$

Furthermore, I assume that the relative quality levels follow the Pareto distribution with power parameter equal to one, an often made assumption in the literature dealing with firm size distribution⁴. This allows me to provide a closed form expression for the mass of the exiting producers:

$$M_t^x = M_t (1 - \chi_{t-1}) \left(1 - \frac{\phi_{t-1}^*}{\phi_t^* \eta_{t-1}} \right) \quad (4.11)$$

⁴Axtell (2001) provides empirical support for this assumption.

4.1.5 Entrants

The mass of potential entrants is assumed to be unbounded, although it will be pinned down by the equilibrium conditions. Similar to incumbents, they engage in R&D activities, although in this case the successful outcome of the innovation process results in entry, rather than an improvement over the existing product.

The entry attempt requires hiring capital and skilled labor, both for the purpose of performing R&D and managerial activities. The cost function mirrors the incumbents' case and the normalized value of entry can be stated as:

$$v_t^e = \max_{\chi_t^e \in [0,1]} \left\{ - \left(1 + \zeta^e r_t^l \right) \omega_t \left(f^e + \frac{1}{a^e} \frac{\chi_t^e}{1 - \chi_t^e} \right) + \chi_t^e \text{E}_t \left[\beta \Lambda_{t,t+1} \left(\frac{Y_{t+1}}{Y_t} \right) v_{t+1} \left(\phi_{t+1}^e \right) \right] \right\}$$

where χ_t^e is the desired entry probability, ζ^e denotes the share of factor rental costs that has to be paid in advance and borrowed at lending rate, f^e is the fixed cost of operation of potential entrants, a^e is the efficiency of R&D inputs in the case of entrants, and ϕ_{t+1}^e represents the expected relative quality level determined upon successful entry.

The first order condition of the potential entrants is given as follows:

$$0 = - \left(1 + \zeta^e r_t^l \right) \frac{\omega_t}{a^e} \frac{1}{(1 - \chi_t^e)^2} + \text{E}_t \left[\beta \Lambda_{t,t+1} \left(\frac{Y_{t+1}}{Y_t} \right) v_{t+1} \left(\phi_{t+1}^e \right) \right] \quad (4.12)$$

The unbounded mass of potential entrants implies that whenever the expected value of entry is positive, more candidates engage in the attempts, driving up the effective costs, and ensuring that the free entry condition holds:

$$v_t^e = 0 \quad (4.13)$$

Following the observations of Acemoglu & Cao (2015) and Garcia-Macia et al. (2016) I assume that entrants enjoy a degree of entry advantage. To account for that and rule out limit pricing in equilibrium, I assume that entrants draw their quality from appropriately upscaled quality distribution of incumbents. As a consequence, the expected relative quality level upon entry is given by:

$$\text{E}_t \left[\phi_{t+1}^e \right] = \frac{\sigma}{\sigma - 1}$$

By denoting with M_t^e the mass of successful entrants, one can pin down the mass of effective potential entrants, which is then given by M_t^e / χ_t^e . Entry is constrained by the supply of skilled resources and is implicitly given by:

$$\frac{(K_t^s)^\alpha (Q_t N_t^s)^{1-\alpha}}{Q_t} = M_t f + (M_t - M_t^x) \left(\frac{1}{a} \frac{\chi_t}{1 - \chi_t} \right) + \frac{M_t^e}{\chi_t^e} \left(f^e + \frac{1}{a^e} \frac{\chi_t^e}{1 - \chi_t^e} \right) \quad (4.14)$$

where the left hand side equals the effective supply of skilled input which is (on the right hand side) split between operating cost of incumbents, R&D activities by continuators, and finally operating cost and R&D activities of potential entrants.

The rate of change in the aggregate quality index depends on the intensity of the R&D activities performed by the incumbents and the mass of new entrants relative to

active establishments. By assuming Pareto distribution of quality levels it is possible to derive the exact closed form expression for the rate of growth of the aggregate quality index:

$$\eta_t = (1 - \chi_t + \chi_{t\ell}) \left(1 - \frac{M_t^e}{M_{t+1}} + \frac{M_t^e}{M_{t+1}} \frac{\sigma}{\sigma - 1} \right) \quad (4.15)$$

Finally, the endogenous probability of incumbent receiving a death shock depends on the exogenous, constant component, and the rate of entry of new establishments that potentially creatively destroy existing establishments. There are three conditions under which an active establishment exits, and I assume that at the end of each period the events follow a specific order. First, the incumbents with relative quality level below ϕ_t^* exit “voluntarily” as their varieties become obsolete. Second, incumbents receive exogenous exit shocks. Finally, a fraction of incumbents are leapfrogged by entrants and thus creatively destroyed. Therefore, the mass of active establishments in the next period is given by:

$$M_{t+1} = M_t - M_t^x - \delta^{exo} (M_t - M_t^x) + [1 - (1 - \delta^{exo}) (M_t - M_t^x)] M_t^e$$

where δ^{exo} is the exogenous exit shock probability and the mass of successful entrants M_t^e is multiplied by the probability that an entrant draws an “unoccupied” location. As by definition creative destruction replaces an incumbent with an entrant, it does not directly affect the mass of active establishments. The expression for active establishment mass can be also written as:

$$M_{t+1} = M_t - M_t^x - \delta_t (M_t - M_t^x) + M_t^e \quad (4.16)$$

Then by comparing the two formulations one gets the following expression for endogenous exit shock probability:

$$\delta_t = 1 - (1 - \delta^{exo}) (1 - M_t^e) \quad (4.17)$$

Intuitively, the probability of not receiving an exit shock is a product of the probabilities of not receiving an exogenous shock and not being creatively destroyed, as the two are independent from each other.

4.1.6 Capital goods producers and the financial system

Perfectly competitive capital goods producers are also the owners of the capital stock which they rent to the establishments. They also borrow from the financial intermediary, at the lending interest rate r_t^l , in order to finance investment in new capital. Therefore, they aim to maximize the expected discounted flow their profits, expressed as:

$$\Pi_t^k = r_t^k K_t - I_t + L_{t+1}^k - (1 + r_t^l) L_t^k$$

where I_t is aggregate investment and L_t^k are loans from the financial intermediary. Physical capital accumulation is subject to the standard constraint:

$$K_{t+1} = I_t + (1 - dp) K_t \quad (4.18)$$

The solution of the capital goods producers' problem yields the following equality between the lending rate and the capital rental rate net of depreciation:

$$r_t^l = r_t^k - dp \quad (4.19)$$

The financial intermediaries collect deposits from households and lend them to two types of entities: intermediate goods producers and potential entrants, and capital producers. The profit of the intermediaries is given by:

$$\Pi_t^f = (1 + r_t^l) L_t^k + (1 + r_t^l) L_t^e - (1 + r_t^d) d_t + d_{t+1} - L_{t+1}^k - L_{t+1}^e$$

where L_t^e denote loans to establishments to finance their working capital requirement, and subject to the loanable funds constraint:

$$L_{t+1}^k + L_{t+1}^e \leq d_{t+1}$$

The financial intermediaries are owned by the households and discount the future in the same manner. Here I make the assumption that the financial intermediaries enjoy a degree of market power that drives a wedge between the deposit and lending interest rates, such that:

$$r_t^l = r_t^d + sp_t \quad (4.20)$$

where sp_t is the spread between the interest rates. While taken literally the variation in spread would imply that the market power of banks is changing over time, it can be also interpreted as a reduced-form way to capture the frictions in the financial markets. The spread is assumed to evolve according to the following AR(1) process:

$$\log sp_t = (1 - \rho_{sp}) \log sp_{ss} + \rho_{sp} \log sp_{t-1} + \varepsilon_{sp,t} \quad (4.21)$$

4.1.7 Market clearing

The markets for factors of production clear:

$$N_t^p = (1 - s) n_t^u \quad \text{and} \quad N_t^s = s n_t^s \quad (4.22)$$

$$K_t = K_t^p + K_t^s \quad (4.23)$$

As all profits are paid to the households, the budget constraint results in the standard resource constraint:

$$Y_t = C_t + I_t + \kappa^u (x_t^u)^2 N_t^p + \kappa^s (x_t^s)^2 N_t^s \quad (4.24)$$

Finally, the process for aggregate productivity is assumed to follow the standard AR(1) form:

$$\log Z_t = \rho_Z \log Z_{t-1} + \varepsilon_{Z,t} \quad (4.25)$$

4.2 Data and results

4.2.1 Data, calibration and estimation

The data used in the Chapter come from several sources. The data on establishment dynamics comes from the US Bureau of Labor Statistics (BLS) Business Employment Dynamics (BDM) database. The data on GDP and its components come from the US Bureau of Economic Analyses (BEA). The data on labor market statistics come predominately from the BLS. To construct the data on vacancies the data from the JOLTS survey, available from December 2000 were spliced with the Composite Help Wanted Index provided by Barnichon (2010). Following Christiano et al. (2014), the series for interest spread was chosen as the Moody's Seasoned BAA Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity, provided by the Federal Reserve Bank of St. Louis, starting in April 1953. The data on R&D spending were taken from the BEA and the National Science Foundation (NSF). The NSF also provides the data on Full-Time Equivalent number of employees performing R&D, although the series ends in 2008.

The model is calibrated to replicate key features of the US economy. The parameters that influence the steady state of the economy are calibrated to reflect the long-run averages in the US data and are summarized in Table 4.1. The average quarterly spread was calculated directly from the corresponding data series. The degree of pre-financing was taken from Christiano et al. (2010). I decided not to differentiate the pre-financing parameters across incumbents and entrants. The values of parameters governing the behavior of the labor markets were taken from previous literature, and their justification was discussed in the previous Chapter.

Both the capital share of income and quarterly depreciation rate are set to values ubiquitous in the business cycle literature. Note that since in the model firms generate positive profits, the labor share of income is lower than $1 - \text{capital share}$. The discount factor, which in the calibration process depends on the value of elasticity of intertemporal substitution, is chosen so that the average annual net deposit interest rate is equal to 4.75%, which together with the assumed average spread implies that the lending rate, equal to the rate of return on capital, equals 6.65%, a value consistent with literature, see e.g. Nishiyama & Smetters (2007). The share of skilled workers is picked to be in the middle of the plausible range of values proposed by Acemoglu et al. (2013) and corresponds to the value used in the previous Chapter. Finally, the set of parameters governing the establishment dynamics is calibrated to match specific moments reported in Table 4.2. As I have 6 moments to match with 8 free parameters, I impose a constraint that the R&D efficiency parameter and fixed cost are equal for both incumbents and entrants.

Table 4.1: Calibrated parameters affecting the steady state

Parameter	Description	Value	Justification
sp_{ss}	Average quarterly spread	0.0047	Annualized spread = 1.9%
ζ, ζ^e	Working capital share	0.75	Christiano et al. (2010)
ρ^u	Unskilled retention rate	0.9725 ³	Cairo & Cajner (2017)
ρ^s	Skilled retention rate	0.99 ³	Cairo & Cajner (2017)
κ^u	Unskilled hiring cost	2	Unskilled job finding probability
κ^s	Skilled hiring cost	15.8	Skilled job finding probability
b^u	Unskilled unemp. benefit	0.14	40% of steady state unskilled wage
b^s	Skilled unemp. benefit	0.41	40% of steady state skilled wage
ψ	Elasticity of matches	0.5	Gertler & Trigari (2009)
σ_m	Matching efficiency	1.7	Average tightness = 0.54
α	Capital share of income	0.3	Standard
dp	Capital depreciation rate	0.025	Standard
β	Discount factor	0.9998	Annual net interest rate of 4.75%
s	Share of skilled workers	0.1039	Chapter 2
ι	Innovative step size	1.016	Annual pc. GDP growth
δ^{exo}	Exog. exit shock prob.	0.0174	Exit rate
a, a^e	R&D efficiency	7.96	Expansions = contractions
f, f^e	Operating fixed cost	0.94	Share of R&D in GDP
θ	Inverse of IES	2.3	Share of investment in GDP
σ	Elasticity of substitution	4.9	Share of R&D employment

Table 4.2: Long-run moments: comparison of model and data

Description	Model	Data	Source
Annual pc. GDP growth	2.06%	2.08%	BEA, 1948q1-2016q4
Exit rate	3.06%	3.07%	BDM, 1992q3-2016q4
Relative share of expanding estabs.	1.01	1.01	BDM, 1992q3-2016q2
Share of R&D in GDP	2.18%	2.23%	BEA, 1948q1-2016q4
Share of investment in GDP	17.50%	17.17%	BEA, 1948q1-2016q4
Share of R&D employment	1.31%	0.98%	NSF & CBP, 1964-2008

To obtain the values of parameters that do not affect the steady state but govern the cyclical behavior of the model, I employ the estimation procedure. The prior distributions were chosen to be relatively uninformative, and in particular the prior distribution for the renegotiation frequency parameter was set to uniform on the unit interval. Table 4.3 in the Appendix contains full information on the priors used.

The estimation makes use of two observable data series. The first one is the growth rate of Real Gross Domestic Product divided by the Labor Force, observed from 1948q2-2017q2. An advantage of the model with explicitly modeled long-run growth is that

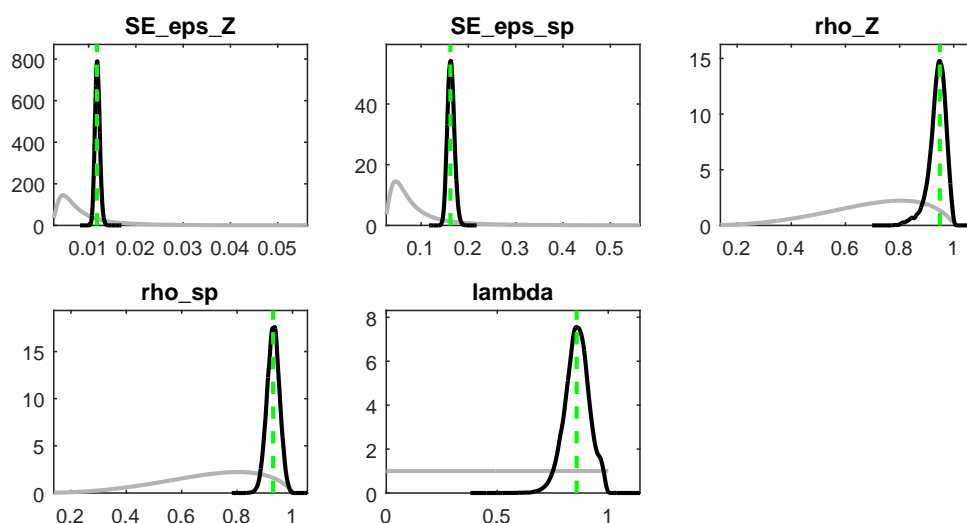
there is no need to detrend the data and valuable information is retained. The second is the demeaned spread between BAA and long-term government bonds. The model was estimated using standard Bayesian procedures with help of Dynare 4.5 and results were generated using two random walk Metropolis-Hastings chains with 200,000 draws each with an acceptance ratio of around 0.24.

Table 4.3 presents the estimation results. The data were clearly informative about the estimated parameters, as the posterior and prior means differ and the highest posterior density (HPD) intervals are relatively tight. This observation can be also confirmed by comparing the plots of prior and posterior densities displayed in Figure 3.1.

Table 4.3: Prior and posterior means of parameters affecting cyclical behavior

Parameter	Description	Prior mean	Post. mean	90% HPD interval
λ	Calvo wage contract prob.	0.5	0.858	[0.766, 0.950]
ρ_Z	Autocorr. of prod. process	0.7	0.939	[0.896, 0.991]
σ_Z	Std. dev. of prod. shock	0.01	0.012	[0.011, 0.013]
ρ_{sp}	Autocorr. of spread process	0.7	0.930	[0.895, 0.968]
σ_{sp}	Std. dev. of spread shock	0.1	0.163	[0.151, 0.174]

Figure 4.1: Prior and posterior distributions of estimated parameters



4.2.2 Model performance and impulse response functions

The data moments were generated on the sample 1948q1-2016q4, with the exception of vacancies and tightness, available from 1951q1, and establishment dynamics, available from 1992q3. The variables trending with population growth, such as GDP and number of establishments, were divided by the Civilian Labor Force and subsequently detrended with Hodrick-Prescott filter.

Table 4.4 presents the comparison of the Hodrick-Prescott filtered moments between the model and data. The upper section of the table is concerned with output and its components, as well as R&D expenditures. The model fits the data very well for output and its components, and only fails to account for much weaker correlation of R&D expenditures with output.

The middle section of the table focuses on variables pertaining to the operations of the labor market. The model wages are stronger correlated with output and have higher autocorrelation than in the data, and model hours are not as volatile as in the data. However, the model is very successful in matching the cyclical behavior of unemployment, vacancies and tightness, achieving nearly perfect fit.

The final section presents the moments related to the establishment dynamics. Although the fit is a bit worse than in the case of previously discussed variables, most of the model moments remain close to their data counterparts, with the exception that the model predicts much smaller volatility of establishment dynamics. The model also predicts that the establishment mass is slightly negatively correlated with output, even though the correlation of net entry with output is almost exactly the same as in the data. A brief look at the impulse response functions in Figure 4.2 reveals that this result is most likely driven by a small and short-lived decrease in the mass of establishments immediately after the shock hits, and for the subsequent periods the mass of active establishments moves in tandem with output. In the case of interest rate spread shocks, Figure 4.3 shows that output and establishment mass comove.

To sum up, although the model is not able to match the data perfectly, the fit is more than satisfactory and provides a solid foundation for further analysis.

Table 4.4: Business cycle moments: comparison of model and data

Variable	Standard deviation		Correlation with Y		Autocorrelation	
	Data	Model	Data	Model	Data	Model
Output	1.58	1.58	1.00	1.00	0.82	0.83
Consumption	0.87	0.69	0.78	0.90	0.82	0.75
Investment	4.54	5.33	0.76	0.97	0.87	0.89
R&D	2.36	2.26	0.32	0.91	0.89	0.92
Wages	0.95	0.73	0.10	0.41	0.68	0.96
Hours	1.36	0.76	0.86	0.92	0.89	0.89
Unemployment	12.76	11.96	-0.77	-0.82	0.89	0.89
Vacancies	13.78	13.47	0.83	0.92	0.91	0.90
Tightness	26.00	24.45	0.82	0.89	0.91	0.91
Establishments	0.62	0.31	0.71	-0.38	0.87	0.89
Expansions	2.84	0.58	0.82	0.78	0.75	0.89
Contractions	2.38	0.76	-0.11	-0.93	0.69	0.89
Net Entry	0.31	0.15	0.31	0.18	0.24	0.45

Figure 4.2 displays the impulse response functions to a standard deviation productivity shock. An increase in productivity raises output both directly and indirectly, through higher investment and hiring rates, which in turn cause an increase in physical capital

stock and hours worked. The response of output to the shock is highly persistent, both due to labor market frictions and the endogenous quality component which permanently shifts output upwards. Expenditures on R&D are also procyclical and persistent.

Staggered wage contract friction prevents wages from responding strongly on impact of the shock, as a large fraction of employment agencies is not allowed to renegotiate wages. Over time, the wage renegotiations take place, and the response of wages exhibits a hump-shaped pattern, reaching the peak at around 3 years after the initial shock. Increased productivity of labor induces the employment agencies to post vacancies, increasing labor market tightness.

Positive productivity shock incentivizes incumbents to increase their R&D expenditures and consequently success probability, and as a consequence the mass of expanding establishments increases, and the mass of contracting establishments decreases. At impact elevated incumbents' demand for scarce resources, especially for skilled labor, results in a temporary decline in net entry rates. However, as more skilled employees become available, net entry turns positive and translates to an increase in the active establishments mass. The rate of growth of aggregate quality index is higher than along the balanced growth path, due to both higher R&D intensity and entry rates. This faster pace of growth is at first maintained by both higher skilled employment and bigger capital stock, although after around 4 years employment returns to its balanced growth path level, leading to a decrease in the rate of quality growth. Nevertheless, more abundant physical capital allows the economy to continue growing faster, and eventually the level of quality relative to balanced growth path trend flattens out and stabilizes at a level around 7% higher than it would be if the shock never happened.

Figure 4.3 displays the impulse response functions to a standard deviation interest spread shock. Broadly speaking, an increase in the wedge between the deposit and lending rates generates effects opposite to the positive productivity shock, and their quantitative size is of order of magnitude smaller. Immediately on impact investment decreases, as it is now more costly to produce new units of physical capital, and consumption rises in response to lower deposit interest rates. Expenditures on R&D initially increase, as incumbents face lower risk of being creatively destroyed due to decreased entry, but after about a year drop below the balanced growth path trend as the recession deepens. Both hours worked and wages decrease in a hump-shaped pattern, while unemployment increases. Creating new vacancies is discouraged, and as a result the labor market becomes less tight. Increased costs of lending deter entry which remains depressed for about 5 years after the initial shock, which also causes a decrease in the mass of establishments. Aggregate quality level remains near its trend level for around 2 years following the shock, as expansions and net entry move in opposite directions. After that period both depressed entry and incumbents' R&D intensity translate to the downward deviation of the quality level from trend, which eventually is lowered by about 0.85% relative to its trend path. Thus the financial shocks, compared to productivity shocks, create similar, although smaller in magnitude, shifts in the balanced growth path of the economy. As a robustness check, Figures A.3 and A.4 in the Appendix presents the Bayesian impulse response functions taking into account parameter uncertainty. All of the results remain unchanged.

Figure 4.2: Impulse response functions to standard deviation productivity shock

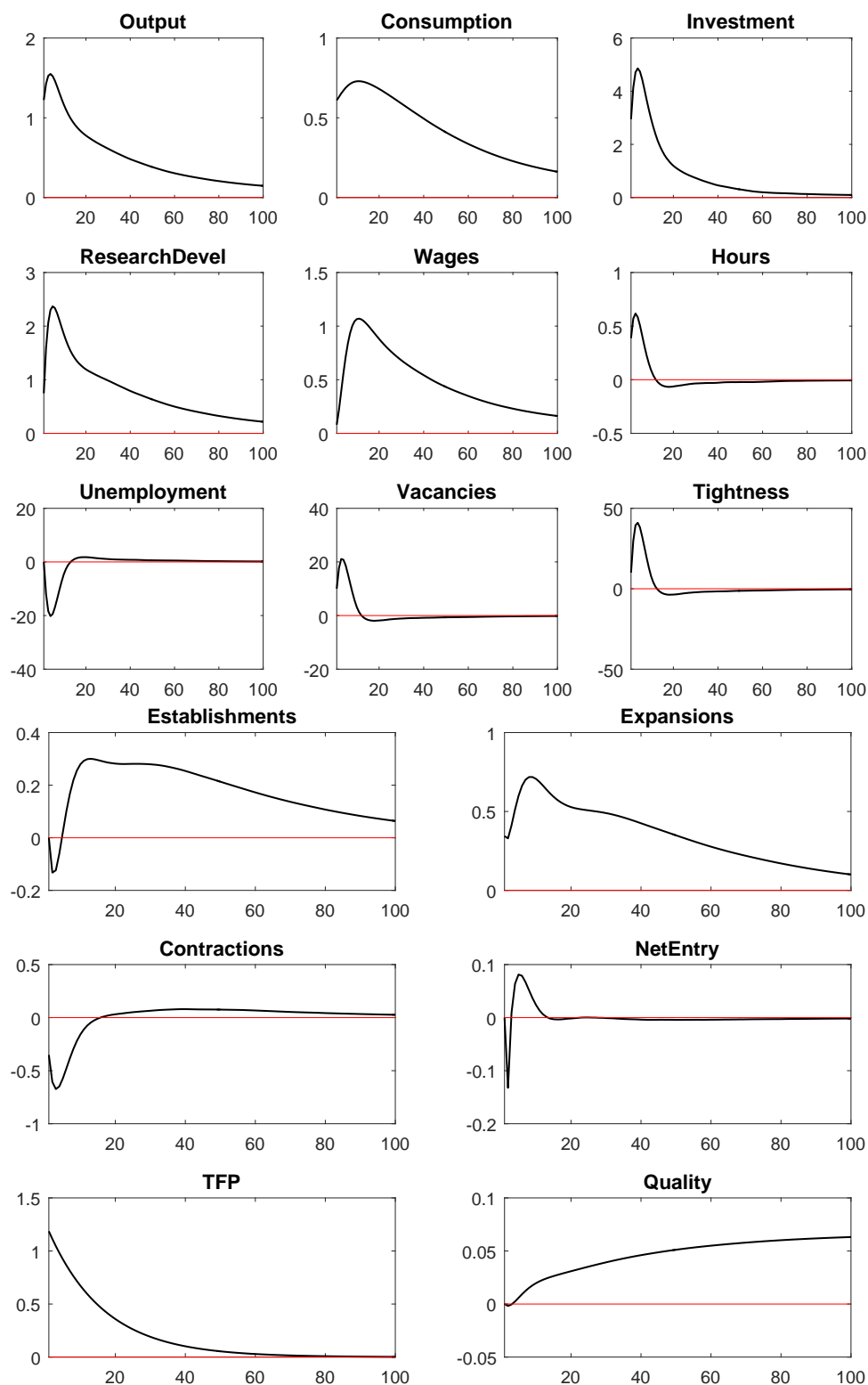
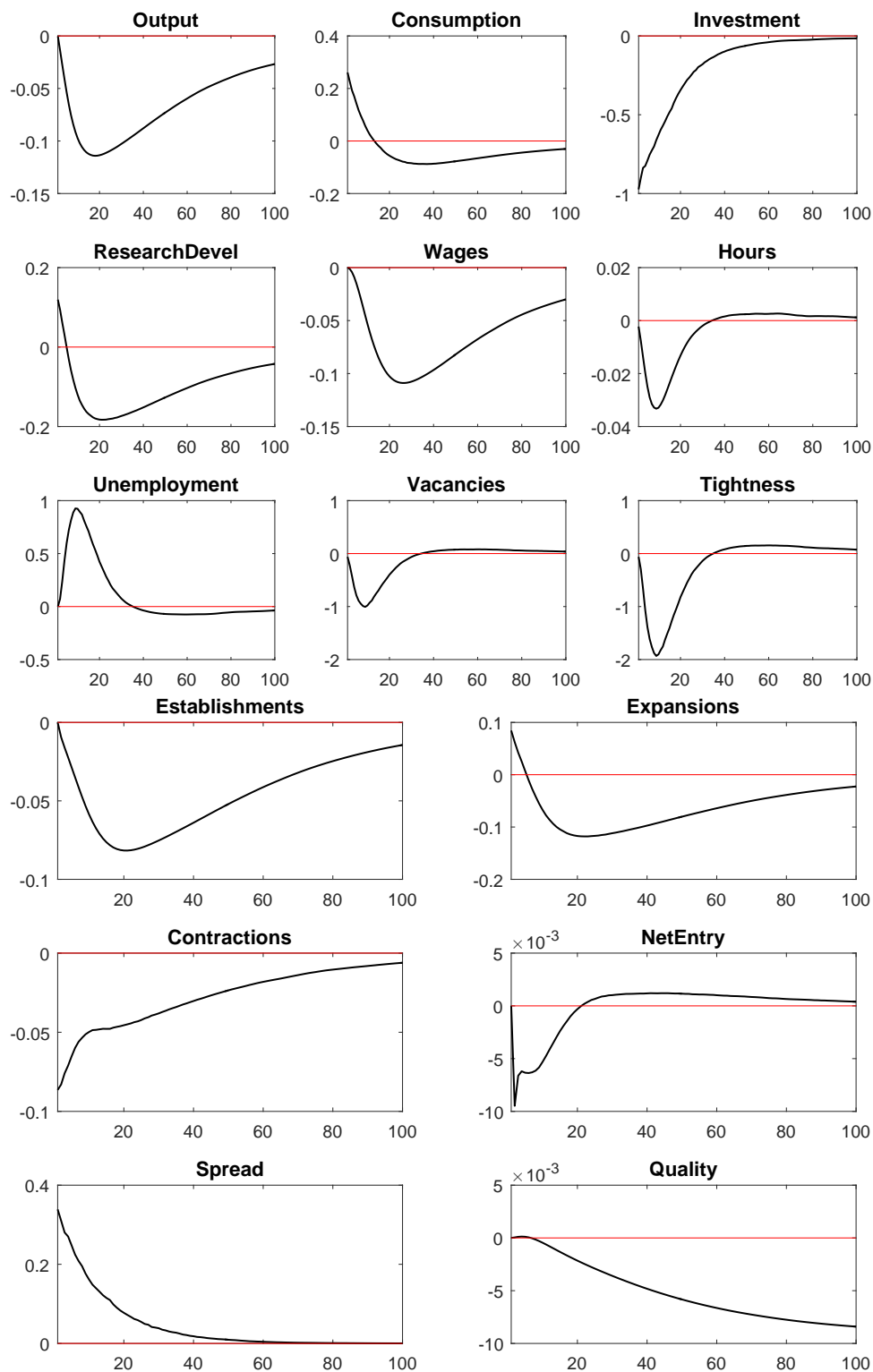


Figure 4.3: Impulse response functions to standard deviation interest rate spread shock



4.3 Long shadows of financial shocks

4.3.1 The experience of Great Recession

The model features mechanisms through which temporary shocks translate to permanent shifts to the balanced growth path of the economy. Therefore it is an attractive laboratory to study the experience of the Great Recession.

The Great Recession has been associated with the largest output drop in the postwar economic history of the United States, which until now remains around 10% below its pre-recession trend. A similar behavior was observed for the R&D expenditures, although the drop was even deeper than for output. Increased establishment exits and depressed entry has resulted in fewer active establishments.

Figure 4.3.1 presents the shock decomposition of key macroeconomic variables since the first quarter of 2000 until the second quarter of 2017. The financial shocks, modeled as increases in the spread between deposit and lending interest rates, account for a nontrivial fraction of the deviation of the variables from their trend. In particular, about a third of the total decline in the establishment mass is attributed to increased spreads, as they are especially harmful to entrants. Depressed entry rates and R&D expenditures result in continuing fall of the aggregate quality index. It has profound implications as while physical capital stock and employment levels can in principle return to their balanced growth path levels, a decline in the aggregate quality is of a more permanent nature and essentially pushes the economy to a balanced growth path below the pre-crisis one.

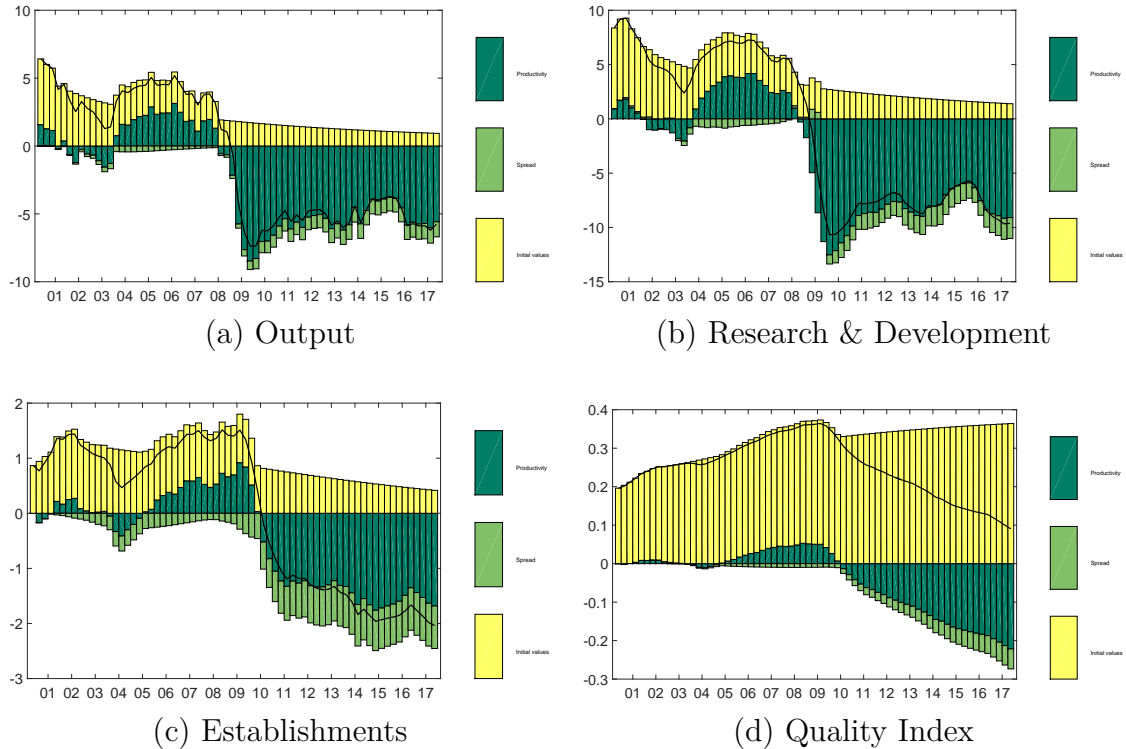


Figure 4.1: Shock decomposition: key macroeconomic variables since 2000

4.3.2 Policy implications

Since temporary shocks can exert level effects on the balanced growth path of the economy, this implies that business cycle fluctuations are associated with additional welfare costs compared to the models where growth results from exogenous processes.

The consumption equivalent method allows to quantify the welfare differences across states of world. The equivalent is equal to the lifetime percentage adjustment in the path of households' consumption that make them indifferent between "living" in two worlds. For the utility function assumed in this Chapter the equivalent-adjusted welfare is given by:

$$W_0(eq) = E_0 \sum_{t=0}^{\infty} \beta^t \frac{((1+eq)c_t)^{1-\theta}}{1-\theta} = (1+eq)^{1-\theta} E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\theta}}{1-\theta}$$

The consumption equivalent across two different worlds can be then computed as follows:

$$eq_{a,b} = \left(\frac{U_0^b}{U_0^a} \right)^{\frac{1}{1-\theta}} - 1$$

where U_0^a and U_0^b denote expected lifetime utilities in worlds a and b , respectively. Then $eq_{a,b}$ has the interpretation of which proportion of consumption the agent living in world a would be willing to forfeit in order to "move" to world b .

Table 4.5 presents the comparison of expected lifetime utilities in five distinct worlds. In the non-stochastic world the economy is not subject to shocks and always remains on its balanced growth path. In the two stochastic worlds the economy is affected by shocks but in the first of them growth is exogenous and the aggregate quality index increases at a constant rate. As a consequence, any welfare losses result from the volatility around the trend and are estimated to be quite low, in accordance with existing literature. The second stochastic world is identical to the model economy. Here welfare losses are significantly larger and stem from the fact that both shocks result in the level shifts of the consumption paths. Finally, the lower section of Table 4.5 is concerned with the relative importance of two shocks for welfare. It turns out that the spread shocks account for about a third of the total welfare costs.

Table 4.5: Welfare cost of business cycles

State of the world	Welfare	Consumption equivalent
Non-stochastic (BGP)	-172.84	—
Stochastic with exogenous growth	-172.97	0.05%
Stochastic with endogenous growth	-191.13	8.04%
Endogenous growth without spread shocks	-178.85	2.66%
Endogenous growth without prod. shocks	-185.52	5.60%

The presence of significant welfare costs of business cycles poses questions on whether economic policy can alleviate some of them. To answer them I examine the macroeconomic and welfare effects of applying several subsidy schemes. Those schemes fall into

two groups: static and countercyclical subsidies. All subsidy schemes are financed via lump-sum taxation.

Static subsidies are designed to act as if a parameter in question was changed by 10%. The direction of change is always in the direction favored by the subsidized agent, e.g. a lowering of operation costs or increasing the R&D efficiency. Table 4.6 documents the results of applying. The first result column displays the average growth rate in stochastic equilibrium. The next two present the extent of change in the aggregate quality index in response to a standard deviation productivity shock, over the horizon of 20 and 100 quarters, respectively, while the following two columns do the same for the spread shock. Next column reports the expected lifetime utility measure, and the following the average unemployment rate in stochastic equilibrium. For ease of interpretation, the last column presents the opposite number to the consumption equivalent, so that the positive value of the statistic indicates welfare gain. As a rule of thumb, *ceteris paribus* households prefer if the aggregate growth rate is higher, volatility (understood as the extent of the reaction of aggregate quality in response to the shock) is lower and unemployment rate is lower.

In agreement with the endogenous growth literature I find that subsidizing R&D expenditures of incumbent establishments is strongly welfare improving, as both the average growth rate is increased and volatility is decreased, at a cost of slightly higher unemployment rate. Welfare gains are also associated with lowering the barriers to entry, either through lowering the fixed costs of prospective entrants or subsidizing their R&D activities. Contrary to the previous literature, e.g. Acemoglu et al. (2013), I find that subsidizing incumbents' operating costs is welfare improving. This discrepancy stems from the fact that although the subsidized economy exhibits lower rate of growth and higher volatility, those effects are dwarfed by gains from decreased churning in the labor market, the full extent of which become apparent only in the stochastic setting.

The lower section of the table presents the effects of subsidies that aim to reduce frictions in the financial markets. Subsidizing the working capital costs of incumbents lowers slightly the volatility of the economy and generates a small welfare gain, while subsidies to working capital of entrants do not have a significant welfare effect. Finally, subsidizing all borrowers in a manner that acts as if the average spread was lower decreases both volatility and average unemployment rate, resulting in welfare gain.

Table 4.6: Effects of static subsidies

	γ	ΔQ_{20}^Z	ΔQ_{100}^Z	ΔQ_{20}^{sp}	ΔQ_{100}^{sp}	U	u	$-eq$
Baseline	2.09	3.17	6.32	0.21	0.82	-191.13	5.65	–
f	2.06	3.33	6.63	0.24	0.87	-187.07	5.53	1.64%
f^e	2.09	3.18	6.36	0.22	0.82	-190.91	5.65	0.09%
a	2.15	3.08	6.12	0.22	0.80	-186.97	5.67	1.68%
a^e	2.10	3.17	6.34	0.22	0.82	-191.05	5.65	0.03%
ζ	2.10	3.16	6.32	0.21	0.82	-190.92	5.65	0.09%
ζ^e	2.10	3.17	6.32	0.21	0.82	-191.15	5.65	-0.01%
sp_{ss}	2.10	3.16	6.31	0.19	0.74	-189.28	5.63	0.75%

Table 4.7 presents the welfare effects of countercyclical subsidies. As they by construc-

tion do not impact significantly the economy's average growth rate or unemployment rate, those variables are not displayed. I consider two variants of subsidies: in the first, if output is observed at level 1% lower than trend, the subsidy increases by 0.5%. Conversely, in the times of boom the subsidy becomes a tax. In the second variant subsidy increases by 5% if the spread is 1 percentage point higher than on average.

The qualitative effects of the two subsidy variants are almost identical, and thus I will discuss only the effects of subsidies reacting to output. Intuitively, subsidy schemes that lower the volatility bring welfare gains. The biggest welfare gains are associated with subsidizing operating costs of active establishments, which gives support for policies aimed at supporting existing firms during recessions. On the other hand, countercyclical subsidies to incumbents' R&D activities are welfare deteriorating, as by redirecting limited resources towards incumbents it exacerbates the difficulties entrants face during downturns. Finally, subsidies to entrants carry small positive welfare gains, while subsidies to working capital have almost no impact on volatility and welfare.

Table 4.7: Effects of countercyclical subsidies

0.5% subsidy if output is 1% below trend						
	ΔQ_{20}^Z	ΔQ_{100}^Z	ΔQ_{20}^{sp}	ΔQ_{100}^{sp}	U	$-eq$
Baseline	3.17	6.32	0.21	0.82	-191.13	–
f	3.11	6.16	0.21	0.79	-187.00	1.67%
f^e	3.16	6.31	0.21	0.82	-190.99	0.06%
a	3.30	6.60	0.22	0.86	-195.82	-1.88%
a^e	3.17	6.32	0.21	0.82	-191.08	0.02%
ζ	3.17	6.32	0.21	0.82	-191.17	-0.01%
ζ^e	3.17	6.32	0.21	0.82	-191.13	0.00%
5% subsidy if spread is 1 percentage point above average						
	ΔQ_{20}^Z	ΔQ_{100}^Z	ΔQ_{20}^{sp}	ΔQ_{100}^{sp}	U	$-eq$
Baseline	3.17	6.32	0.21	0.82	-191.13	–
f	3.17	6.32	0.20	0.79	-189.28	0.75%
f^e	3.17	6.32	0.21	0.82	-191.07	0.02%
a	3.17	6.32	0.26	0.88	-193.20	-0.83%
a^e	3.17	6.32	0.21	0.82	-191.11	0.01%
ζ	3.17	6.32	0.21	0.82	-191.15	-0.00%
ζ^e	3.17	6.32	0.21	0.82	-191.13	0.00%

4.4 Conclusions

The Great Recession has resulted in a seemingly permanent level shift in many macroeconomic variables. This Chapter has presented an endogenous growth model where monopolistically competitive, heterogeneous establishments choose the level of R&D expenditures. The model economy is also subject to the search and matching friction in the labor market, as well as financial friction modeled as a reduced-form shock to the spread

between deposit and lending interest rates. This setup generates volatile and procyclical R&D expenditure patterns and is consistent with the business cycle dynamics of GDP and its components, labor market variables, as well as establishment dynamics.

I find that both productivity and financial shocks affect the endogenous growth rate of the economy, resulting in level shifts in the balanced growth path. This significantly increases the estimate of the welfare costs of business cycles. As a consequence, economic policy can play an important role in alleviating the consequences of those shocks. I analyze the macroeconomic and welfare effects of a series of static and countercyclical subsidy schemes.

Regarding the static subsidies, I find that subsidizing R&D expenditures, as well as lowering barriers to entry, is welfare improving, in line with endogenous growth literature. At odds with this literature, static subsidies to incumbents' operating costs are also found to be welfare improving. This result stems from taking into account the effects of business cycle fluctuations in an economy with frictional labor and financial markets.

Regarding the countercyclical subsidies, I find that subsidizing R&D expenditures of active establishments is welfare deteriorating, as it redirects precious resources from more efficient uses. On the other hand, subsidizing incumbents' operating costs is welfare enhancing, regardless of whether the economy is hit by productivity or financial shock. This result supports implementing policies that aim to reduce exits of active establishments during recessions.

Chapter 5

Conclusions and policy recommendations

How (and if) should we foster innovations? Should we reduce entry barriers for new firms, or maybe should we subsidize firms investing in research and development? What are the consequences of those policies on economic growth rate and the effects of business cycles? Finally, are active labor market policies conducive for economic growth and innovativeness?

Economic literature has attempted to answer the above, and similar, questions. However, existing studies usually had to pick between analyzing those issues from the vantage points of either endogenous economic growth, or business cycles. My PhD thesis adopts a novel, tractable approach that enables to analyze the effects of aforementioned policies in a model featuring both endogenous economic growth and business cycle fluctuations, as well as generating establishment dynamics in line with the data. Thanks to that the results are more robust and it is possible to analyze the effects of additional policies that are absent in the extant literature.

I have built a model in which heterogeneous, monopolistically competitive firms decide on their outlays on research and development activities, depending on the degree of competition and the business cycle phase. The R&D intensity in turn determines the rate of arrival of innovations and in consequence the rate of endogenous growth. By using this model and the United States data on macroeconomic variables for years 1948-2017, as well as data on establishment dynamics for years 1992-2017, I was able to conclude the following:

1. Business cycles of amplitude observed in the United States have a slightly positive effect on the average annual rate of growth (increasing it from 2.02% to 2.04%). Despite this higher rate of growth, the welfare effects of business cycles are of two orders of magnitude higher than in the traditional business cycle studies. This implies that the households would prefer to live in the world without fluctuations. [Chapter 2]
2. In line with existing endogenous growth literature I find that the barriers to entry and cost of innovation are important in determining the rate of growth and impact welfare. I conclude that it would be welfare improving to lower the barriers to entry and subsidize research and development activities. Taking business cycle

fluctuations into account allows to analyze countercyclical subsidy schemes, that is subsidies that are active during recessions. It turns out that countercyclical subsidies for existing firms are welfare improving, which is a novel result. [Chapter 2]

3. The above results are qualitatively unchanged in the extended versions of the model which take into account labor market frictions, as well as financial market frictions and shocks. Under these conditions it turns out that it is welfare improving to subsidize incumbent firms, which is a novel result. Despite lowering the average rate of growth (from 2.09% to 2.05-2.06%), the effects of business cycles are moderated and the labor markets are stabilized, resulting in lower average rate of unemployment (5.2-5.5% relative to 5.7% without subsidies). [Chapters 3 and 4]
4. Active labor market policies do not affect perceptibly the rate of economic growth, but are clearly welfare improving. Lowering the costs in the search and matching process yields welfare increase equivalent to increasing consumption by about 0.75%, which is supportive for the flexicurity model of the labor market. [Chapter 3]
5. Taking into account financial market frictions and shocks significantly increases the welfare costs of business cycles. It turns out that they may be responsible for around 1/3 of total welfare costs. It is then important to aim to reduce average spread between the borrowing and lending interest rates, as well as mitigate the shocks to spread. [Chapter 4]

As I am not the first to analyze the effects of subsidy schemes within the endogenous growth framework, a discussion of similarities and dissimilarities with existing literature is in order. The effects of static subsidies were analyzed exhaustively in the literature, and have received textbook treatments in e.g. Aghion & Howitt (1998), Barro & Sala-i Martin (2004), Aghion & Howitt (2008) and Acemoglu (2009). A ubiquitous result in the neo-Schumpeterian quality-ladder framework is that subsidies incentivizing higher R&D spending are welfare improving, at least when the business stealing effect is small or properly internalized. As in my dissertation growth results from innovations by both incumbents and entrants, the business stealing effect does not dominate the results, and I also find a welfare enhancing role of R&D subsidies. Another often reached conclusion is that subsidies to the operating costs of incumbents are welfare deteriorating, as they effectively increase barriers to entry, see e.g. Acemoglu et al. (2013). I also obtain this result in the stochastic setting. However, the results from the model extended by including frictions in the labor markets overturn this conclusion, as the welfare costs of labor market turnover are substantial. As a consequence, static subsidies to incumbents, while lowering the average rate of growth, decrease the average rate of unemployment, generating positive level effects. This result could not be obtained in a purely deterministic model, and this is where I contribute the most to the existing literature.

Concerning countercyclical subsidies, Nuño (2011) considers a stochastic extension of the Aghion & Howitt (1998) model and finds that countercyclical subsidies to R&D have no positive effect on welfare. In his setup, growth results from entrants leapfrogging over incumbents. In this dissertation I consider a model which allows for innovation by both incumbents and entrants, and find that indeed the welfare effects of countercyclical R&D subsidies to entrants are positive but minuscule. On the other hand, countercyclical

subsidies to incumbents' R&D are strongly welfare deteriorating, as such subsidies redirect precious R&D resources away from the entrants at times where the entry rates are already depressed. This “crowding out” effect is ignored by the literature that focuses solely on the impact of R&D subsidies on the performance of the incumbents, e.g. Brautzsch et al. (2015).

I also consider the effects of countercyclical subsidies to both incumbents' and entrants' operating costs. The results from all variants of the model imply that countercyclical subsidies to incumbents are strongly welfare improving, as they boost R&D expenditure during recessions and thus mitigate the negative effects of business cycle fluctuations on the endogenous rate of growth, while generating smaller “crowding out” effects on the R&D expenditures of entrants.

To summarize, while more research on the macroeconomic and welfare effects of countercyclical subsidies is needed, the results of this dissertation imply that implementing static subsidies to R&D expenditures and supporting existing establishments and firms during severe downturns may constitute an optimal endogenous growth policy.

Appendix A

Appendices

A.1 Chapter 2

A.1.1 Formula for real operating profit

Real operating profit:

$$\begin{aligned}\pi_t^o(i) &= p_t(i) y_t(i) - w_t^u n_t^u(i) - w_t^s f \\ &= p_t(i) y_t(i) - mc_t(i) y_t(i) - w_t^s f \\ &= p_t(i) y_t(i) - \frac{\sigma-1}{\sigma} p_t(i) y_t(i) - w_t^s f \\ &= \left(1 - \frac{\sigma-1}{\sigma}\right) p_t(i) Y_t p_t(i)^{-\sigma} - w_t^s f \\ &= \frac{1}{\sigma} Y_t \left(\frac{\sigma}{\sigma-1} \frac{w_t^u}{Z_t q_t(i)}\right)^{1-\sigma} - w_t^s f \\ &= \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} Y_t (w_t^u)^{1-\sigma} Z_t^{\sigma-1} q_t(i)^{\sigma-1} - w_t^s f\end{aligned}\tag{A.1.1}$$

Aggregate price index:

$$\begin{aligned}P_t &= \left[\int_0^{M_t} P_t(i)^{1-\sigma} di \right]^{\frac{1}{1-\sigma}} \\ &= \left[M_t \int_0^\infty P_t(q)^{1-\sigma} \mu_t(q) dq \right]^{\frac{1}{1-\sigma}} \\ &= M_t^{\frac{1}{1-\sigma}} \left[\int_0^\infty \left(\frac{\sigma}{\sigma-1} \frac{P_t w_t^u}{Z_t q} \right)^{1-\sigma} \mu_t(q) dq \right]^{\frac{1}{1-\sigma}} \\ &= M_t^{\frac{1}{1-\sigma}} \frac{\sigma}{\sigma-1} \frac{P_t w_t^u}{Z_t} \left[\int_0^\infty q^{\sigma-1} \mu_t(q) dq \right]^{\frac{1}{\sigma-1}}^{-1} \\ &= M_t^{\frac{1}{1-\sigma}} \frac{\sigma}{\sigma-1} \frac{P_t w_t^u}{Z_t Q_t}\end{aligned}\tag{A.1.2}$$

Real unskilled wage:

$$w_t^u = \frac{\sigma-1}{\sigma} Z_t Q_t M_t^{\frac{1}{\sigma-1}}\tag{A.1.3}$$

Real operating profit again:

$$\begin{aligned}
\pi_t^o(i) &= \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} Y_t (w_t^u)^{1-\sigma} Z_t^{\sigma-1} q_t(i)^{\sigma-1} - w_t^s f \\
&= \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} Y_t \left(M_t^{\frac{1}{\sigma-1}} \frac{\sigma-1}{\sigma} Z_t Q_t \right)^{1-\sigma} Z_t^{\sigma-1} q_t(i)^{\sigma-1} - w_t^s f \\
&= \frac{1}{\sigma} \frac{Y_t}{M_t} (q_t(i)/Q_t)^{\sigma-1} - w_t^s f \\
&= \frac{1}{\sigma} \frac{Y_t}{M_t} \phi_t(i) - w_t^s f
\end{aligned} \tag{A.1.4}$$

A.1.2 Formula for aggregate output

Relative labor input:

$$\begin{aligned}
\frac{y_t(i)}{y_t(j)} &= \frac{y_t p_t(i)^{-\sigma}}{y_t p_t(j)^{-\sigma}} = \left(\frac{\frac{\sigma}{\sigma-1} \frac{w_t^u}{z_t q_t(i)}}{\frac{\sigma}{\sigma-1} \frac{w_t^u}{z_t q_t(j)}} \right)^{-\sigma} = \left(\frac{q_t(i)}{q_t(j)} \right)^\sigma \\
\frac{Z_t q_t(i) n_t^u(i)}{Z_t q_t(j) n_t^u(j)} &= \left(\frac{q_t(i)}{q_t(j)} \right)^\sigma \\
\frac{n_t^u(i)}{n_t^u(j)} &= \left(\frac{q_t(i)}{q_t(j)} \right)^{\sigma-1} \longrightarrow n_t^u(i) = \left(\frac{q_t(i)}{Q_t} \right)^{\sigma-1} n_t^u(Q_t)
\end{aligned}$$

where $n_t^u(Q_t)$ is the level of employment by establishment with quality level equal to the aggregate quality index.

Aggregate labor input:

$$\begin{aligned}
N_t^u &= \left[\int_0^{M_t} n_t^u(i) di \right] \\
&= \left[M_t \int_0^\infty n_t^u(q) \mu_t(q) dq \right] \\
&= M_t \left[\int_0^\infty \left(\frac{q_t(i)}{Q_t} \right)^{\sigma-1} n_t^u(Q_t) \mu_t(q) dq \right] \\
&= M_t Q_t^{1-\sigma} n_t^u(Q_t) \left[\int_0^\infty q_t(i)^{\sigma-1} \mu_t(q) dq \right] \\
&= M_t Q_t^{1-\sigma} n_t^u(Q_t) Q_t^{\sigma-1} \\
&= M_t n_t^u(Q_t)
\end{aligned}$$

$$n_t^u(Q_t) = \frac{N_t^u}{M_t} \tag{A.1.5}$$

Aggregate output:

$$\begin{aligned}
Y_t &= \left[\int_0^{M_t} y_t(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}} \\
&= \left[M_t \int_0^\infty y_t(q)^{\frac{\sigma-1}{\sigma}} \mu_t(q) dq \right]^{\frac{\sigma}{\sigma-1}} \\
&= M_t^{\frac{\sigma}{\sigma-1}} \left[\int_0^\infty (Z_t q n_t^u(q))^{\frac{\sigma-1}{\sigma}} \mu_t(q) dq \right]^{\frac{\sigma}{\sigma-1}} \\
&= M_t^{\frac{\sigma}{\sigma-1}} Z_t \left[\int_0^\infty \left(q \left(\frac{q}{Q_t} \right)^{\sigma-1} \frac{N_t^u}{M_t} \right)^{\frac{\sigma-1}{\sigma}} \mu_t(q) dq \right]^{\frac{\sigma}{\sigma-1}} \\
&= M_t^{\frac{\sigma}{\sigma-1}-1} Z_t Q_t^{1-\sigma} N_t^u \left[\int_0^\infty (q_t^\sigma)^{\frac{\sigma-1}{\sigma}} \mu_t(q) dq \right]^{\frac{\sigma}{\sigma-1}} \\
&= M_t^{\frac{1}{\sigma-1}} Z_t Q_t^{1-\sigma} N_t^u \left[\left[\int_0^\infty q_t(i)^{\sigma-1} \mu_t(q) dq \right]^{\frac{1}{\sigma-1}} \right]^\sigma \\
&= M_t^{\frac{1}{\sigma-1}} Z_t Q_t^{1-\sigma} N_t^u Q_t^\sigma \\
&= Z_t Q_t M_t^{\frac{1}{\sigma-1}} N_t^u
\end{aligned} \tag{A.1.6}$$

A.1.3 Aggregate quality evolution

Aggregate quality index at the end of period t :

$$Q_t = \left[\int_0^\infty q^{\sigma-1} \mu_t(q) dq \right]^{\frac{1}{\sigma-1}} = \left[\frac{1}{1 - G_t(q_{t-1}^*)} \int_{q_{t-1}^*}^\infty q^{\sigma-1} g_t(q) dq \right]^{\frac{1}{\sigma-1}} \quad (\text{A.1.7})$$

The aggregate quality level after exits and innovation resolution but before entry:

$$\begin{aligned} Q_t^* &= \left\{ \frac{1}{1 - G_t(q_t^*)} \left[(1 - \chi_t) \int_{q_t^*}^\infty q^{\sigma-1} g_t(q) dq + \chi_t \int_{q_t^*}^\infty \left(\iota^{\frac{1}{\sigma-1}} q \right)^{\sigma-1} g_t(q) dq \right] \right\}^{\frac{1}{\sigma-1}} \\ &= \left[(1 - \chi_t + \chi_t \iota) \frac{1}{1 - G_t(q_t^*)} \int_{q_t^*}^\infty q^{\sigma-1} g_t(q) dq \right]^{\frac{1}{\sigma-1}} \end{aligned} \quad (\text{A.1.8})$$

Aggregate quality index in $t + 1$ after entry:

$$\begin{aligned} Q_{t+1} &= \left\{ \frac{1 - \chi_t + \chi_t \iota}{1 - G_t(q_t^*)} \left[\left(1 - \frac{M_t^e}{M_{t+1}} \right) \int_{q_t^*}^\infty q^{\sigma-1} g_t(q) dq + \frac{M_t^e}{M_{t+1}} \int_{q_t^*}^\infty \left(\left(\frac{\sigma}{\sigma-1} \right)^{\frac{1}{\sigma-1}} q \right)^{\sigma-1} g_t(q) dq \right] \right\}^{\frac{1}{\sigma-1}} \\ &= \left[(1 - \chi_t + \chi_t \iota) \left(1 - \frac{M_t^e}{M_{t+1}} + \frac{M_t^e}{M_{t+1}} \frac{\sigma}{\sigma-1} \right) \frac{1}{1 - G_t(q_t^*)} \int_{q_t^*}^\infty q^{\sigma-1} g_t(q) dq \right]^{\frac{1}{\sigma-1}} \end{aligned} \quad (\text{A.1.9})$$

Transformed aggregate growth rate η_t :

$$\begin{aligned} \eta_t &= \left(\frac{Q_{t+1}}{Q_t} \right)^{\sigma-1} \\ &= \left\{ \frac{\left[(1 - \chi_t + \chi_t \iota) \left(1 - \frac{M_t^e}{M_{t+1}} + \frac{M_t^e}{M_{t+1}} \frac{\sigma}{\sigma-1} \right) \frac{1}{1 - G_t(q_t^*)} \int_{q_t^*}^\infty q^{\sigma-1} g_t(q) dq \right]^{\frac{1}{\sigma-1}}}{\left[\frac{1}{1 - G_t(q_{t-1}^*)} \int_{q_{t-1}^*}^\infty q^{\sigma-1} g_t(q) dq \right]^{\frac{1}{\sigma-1}}} \right\}^{\sigma-1} \\ &\approx (1 - \chi_t + \chi_t \iota) \left(1 - \frac{M_t^e}{M_{t+1}} + \frac{M_t^e}{M_{t+1}} \frac{\sigma}{\sigma-1} \right) \end{aligned} \quad (\text{A.1.10})$$

where if the distribution is invariant with respect to the cutoff points q_{t-1}^* and q_t^* (as is the case with Pareto and other power-law distributions) then the above relationship holds with equality.

A.1.4 Labor market equilibrium

Unskilled consumption-labor choice:

$$\begin{aligned}
(1-s) \psi_t^u (n_t^u)^\kappa &= w_t^u c_t^{-\theta} \\
(1-s) (\psi_t^u Q_t^{1-\theta}) (n_t^u)^\kappa &= \left(\frac{\sigma-1}{\sigma} Q_t Z_t M_t^{\frac{1}{\sigma-1}} \right) \left(Q_t Z_t M_t^{\frac{1}{\sigma-1}} (1-s) n_t^u \right)^{-\theta} \\
(1-s) \psi_t^u (n_t^u)^{\kappa+\theta} &= \frac{\sigma-1}{\sigma} Z_t^{1-\theta} M_t^{\frac{1-\theta}{\sigma-1}} (1-s)^{-\theta} \\
n_t^u &= \left[\frac{\sigma-1}{\sigma} Z_t^{1-\theta} M_t^{\frac{1-\theta}{\sigma-1}} (1-s)^{-\theta-1} / \psi_t^u \right]^{\frac{1}{\kappa+\theta}} \tag{A.1.11}
\end{aligned}$$

Skilled consumption-labor choice:

$$\begin{aligned}
s \psi_t^s (n_t^s)^\kappa &= w_t^s c_t^{-\theta} \\
s (\psi_t^s Q_t^{1-\theta}) (n_t^s)^\kappa &= \left(\omega_t Q_t Z_t M_t^{\frac{1}{\sigma-1}} N_t^u \right) \left(Q_t Z_t M_t^{\frac{1}{\sigma-1}} N_t^u \right)^{-\theta} \\
s \psi_t^s (n_t^s)^\kappa &= \omega_t \left(Z_t M_t^{\frac{1}{\sigma-1}} N_t^u \right)^{1-\theta} \\
n_t^s &= \left[\omega_t \left(Z_t M_t^{\frac{1}{\sigma-1}} N_t^u \right)^{1-\theta} / (s \psi_t^s) \right]^{\frac{1}{\kappa}} \tag{A.1.12}
\end{aligned}$$

A.1.5 Shape of the relative quality distribution

Consider the situation of incumbents with relative quality levels $\phi > \phi_t^*$ such that they will all choose the same success probability α . Therefore, for all of them:

$$\phi_{t+1} = \begin{cases} \iota \phi_t / \eta_t & \text{with probability } \alpha_t \\ \phi_t / \eta_t & \text{with probability } 1 - \alpha_t \end{cases} \tag{A.1.13}$$

Define the counter-cumulative distribution of relative establishment quality by $H_t(\varphi) = \mathbb{P}(\phi_t > \varphi)$. The equation for motion of H is given by:

$$H_{t+1} \left(\frac{\varphi}{\zeta_t} \right) = \alpha_t H_t \left(\frac{\eta_t \varphi}{\iota} \right) + (1 - \alpha_t) H_t \left(\frac{\varphi}{\iota} \right) \quad \text{for } \varphi > \phi_t^* \tag{A.1.14}$$

where $\zeta_t \equiv [\chi_t (\iota - 1) + 1] / \eta_t$ takes into account that an incumbent in expectation “slides down” in the relative quality distribution due to presence of entry advantage.

Conjecture that the counter-cumulative distribution takes the power-law form $H_t(\varphi) = \Gamma\varphi^{-k}$. Under this conjecture, the ergodic distribution satisfies:

$$\begin{aligned}\frac{\Gamma}{\left(\frac{\varphi}{\zeta_t}\right)^k} &= \frac{\chi_t\Gamma}{\left(\frac{\eta_t\varphi}{\iota}\right)^k} + \frac{(1-\chi_t)\Gamma}{(\eta_t\varphi)^k} \\ \zeta_t^k &= \frac{\chi_t}{\left(\frac{\eta_t}{\iota}\right)^k} + \frac{(1-\chi_t)}{(\eta_t)^k} \\ (\zeta_t\eta_t)^k &= \iota^k\chi_t + (1-\chi_t)\end{aligned}\tag{A.1.15}$$

Only two values of k can satisfy the above equation: $k = 0$ and $k = 1$. The first case is a degenerate one where the distribution collapses to a point mass, which we disregard. Therefore, the only other possibility is that the BGP distribution is Pareto with shape parameter $k = 1$ and scale parameter $\Gamma = \phi^*$:

$$\mathrm{P}(\varphi \leq \phi) = 1 - \frac{\phi^*}{\phi}\tag{A.1.16}$$

Note however, that this conclusion only applies to the right of ϕ^* , that is, we know that the distribution possesses a Pareto right tail. As the establishments located to the left of ϕ^* will exit anyway and do not invest in R&D, the effects on aggregate growth rate from this portion of the distribution are negligible and I opt to approximate the “true” relative quality distribution with Pareto distribution for the entire support.

A.1.6 Solution of incumbents' problem along the BGP

First order and envelope conditions of incumbents:

$$B = \frac{\omega}{a} \frac{1}{(1-\chi)^2} \frac{1}{\xi} \frac{\eta}{\iota-1}$$

$$B = \left(\frac{1}{\sigma M} - \frac{\omega}{a} \frac{\chi}{1-\chi} \right) + \xi B \frac{\alpha(\iota-1)+1}{\eta}$$

Use $\zeta = [\chi(\iota-1)+1]/\eta$ to simplify the envelope condition:

$$B = \frac{\frac{1}{\sigma M} - \frac{\omega}{a} \frac{\chi}{1-\chi}}{1 - \xi \zeta} \quad (\text{A.1.17})$$

A.1.6.1 “Partial equilibrium” approach

Rework the first order condition:

$$B = \frac{\omega}{a} \frac{1}{(1-\chi)^2} \frac{1}{\xi} \frac{\eta}{\iota-1} = \frac{\omega}{a} \frac{1}{\xi} \frac{1}{1-\chi} \frac{\eta}{\iota - [\chi(\iota-1)-1]} = \frac{\omega}{a} \frac{1}{\xi} \frac{1}{1-\chi} \frac{\eta}{\iota - \zeta \eta}$$

Equate both expressions for B :

$$\begin{aligned} \frac{\frac{1}{\sigma M} - \frac{\omega}{a} \frac{\chi}{1-\chi}}{1 - \xi \zeta} &= \frac{\omega}{a} \frac{1}{\xi} \frac{1}{1-\chi} \frac{\eta}{\iota - \zeta \eta} \\ \frac{1}{\sigma M} - \frac{\omega}{a} \frac{\chi}{1-\chi} &= \frac{\omega}{a} \frac{1 - \xi \zeta}{\xi} \frac{1}{1-\chi} \frac{\eta}{\iota - \zeta \eta} \quad | \quad \cdot \frac{a}{\omega} (1-\chi) \\ \frac{1}{\sigma M} \frac{a}{\omega} (1-\chi) - \chi &= \frac{1 - \xi \zeta}{\xi} \frac{\eta}{\iota - \zeta \eta} \\ \frac{1}{\sigma M} \frac{a}{\omega} - \chi \left(1 + \frac{1}{\sigma M} \frac{a}{\omega} \right) &= \frac{1 - \xi \zeta}{\xi} \frac{\eta}{\iota - \zeta \eta} \\ \chi &= \frac{\frac{1}{\sigma M} \frac{a}{\omega} - \frac{1 - \xi \zeta}{\xi} \frac{\eta}{\iota - \zeta \eta}}{1 + \frac{1}{\sigma M} \frac{a}{\omega}} \end{aligned} \quad (\text{A.1.18})$$

Solution exists if:

$$\chi \in [0, 1] \iff \frac{1}{\sigma M} \frac{a}{\omega} \geq \frac{1 - \xi \zeta}{\xi} \frac{\eta}{\iota - \zeta \eta} \quad (\text{A.1.19})$$

A.1.6.2 “Explicit solution” approach

Equate both expressions for B :

$$\begin{aligned}\frac{\frac{1}{\sigma M} - \frac{\omega}{a} \frac{\chi}{1-\chi}}{1 - \xi \zeta} &= \frac{\omega}{a} \frac{1}{(1-\chi)^2} \frac{1}{\xi} \frac{\eta}{\iota - 1} \\ \frac{1}{\sigma M} - \frac{\omega}{a} \frac{\chi}{1-\chi} &= \frac{\omega}{a} \frac{1 - \xi \zeta}{\xi} \frac{1}{(1-\chi)^2} \frac{\eta}{\iota - 1} \\ \frac{1}{\sigma M} - \frac{\omega}{a} \frac{\chi}{1-\chi} &= \frac{\omega}{a} \frac{1 - \xi \zeta}{\xi} \frac{1}{(1-\chi)^2} \frac{\eta}{\iota - 1} \\ \frac{1}{\sigma M} \frac{a}{\omega} (1 - \alpha)^2 \xi (\iota - 1) - \chi (1 - \chi) \xi (\iota - 1) &= \left(1 - \xi \frac{\chi (\iota - 1) + 1}{\eta}\right) \eta\end{aligned}$$

Define:

$$X \equiv \frac{1}{\sigma M} \frac{a}{\omega} \xi (\iota - 1) \quad (\text{A.1.20})$$

$$\begin{aligned}X (1 - 2\chi + \chi^2) - (\chi - \chi^2) \xi (\iota - 1) &= \eta - \xi (\iota - 1) \chi - \xi \\ \chi^2 (X + \xi (\iota - 1)) + \chi (-2X - \xi (\iota - 1) + \xi (\iota - 1)) + (X + \xi - \eta) &= 0 \\ \chi^2 (X + \xi (\iota - 1)) + \chi (-2X) + (X + \xi - \eta) &= 0\end{aligned}$$

Solve:

$$\chi = \frac{X \pm \sqrt{X^2 - (X + \xi (\iota - 1)) (X + \xi - \eta)}}{(X + \xi (\iota - 1))} \quad (\text{A.1.21})$$

Existence of real roots is assured:

$$\begin{aligned}X^2 - (X + \xi (\iota - 1)) (X + \xi - \eta) &> 0 \\ X^2 - \left(X^2 + X (\xi - \eta) + X \xi (\iota - 1) + \xi (\iota - 1) (\xi - \eta) \right) &> 0 \\ - \underbrace{(X (\xi \iota - \eta))}_{<0} + \underbrace{\xi (\iota - 1)}_{>0} \underbrace{(\xi - \eta)}_{<0} &> 0\end{aligned} \quad (\text{A.1.22})$$

since $\iota > \xi \iota > \eta > 1 > \xi$.

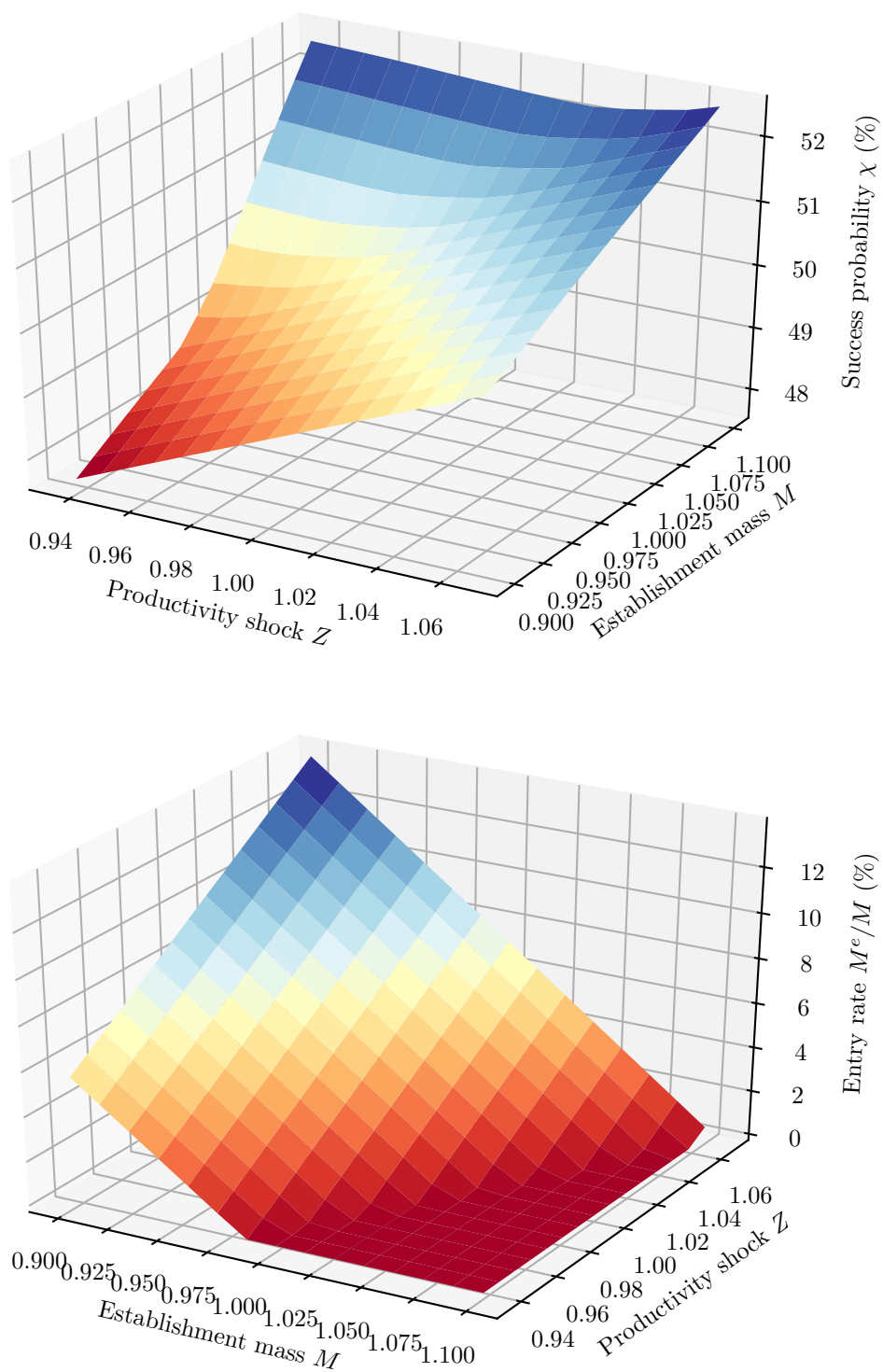
Uniqueness is also almost certain. Consider :

$$\begin{aligned}
& \frac{X + \sqrt{X^2 - (X + \xi (\iota - 1)) (X + \xi - \eta)}}{(X + \xi (\iota - 1))} > 1 \\
& X + \sqrt{X^2 - (X + \xi (\iota - 1)) (X + \xi - \eta)} > X + \xi (\iota - 1) \\
& \sqrt{-(X (\xi \iota - \eta) + \xi (\iota - 1) (\xi - \eta))} > \xi (\iota - 1) \quad | \quad .^2 \\
& -(\underbrace{X (\xi \iota - \eta)}_{<0} + \underbrace{\xi (\iota - 1)}_{>0} \underbrace{(\xi - \eta)}_{<0}) > \xi^2 (\iota - 1)^2 \approx 0 \tag{A.1.23}
\end{aligned}$$

We know from Equation A.1.22 that the LHS of above equation is positive. The above inequality is likely to hold since $(\iota - 1)^2 \approx 0$.

A.1.7 Additional figures

Figure A.1: Chapter 2: Policy functions of incumbents and entrants, 3d graphs



A.2 Chapter 3

A.2.1 Additional derivations

A.2.1.1 Solutions of cost minimization problems

Intermediate goods production sector

$$\begin{aligned} \min \quad & tc_t^p(i) = \tilde{w}_t^u n_t^p(i) + r_t k_t^p(i) \\ \text{subject to} \quad & y_t(i) = Z_t k_t^p(i)^\alpha [q_t(i) n_t^p(i)]^{1-\alpha} \end{aligned}$$

FOCs

$$\begin{aligned} n_t(i) &: \tilde{w}_t^u = \lambda^p (1 - \alpha) Z_t k_t^p(i)^\alpha q_t(i)^{1-\alpha} n_t^p(i)^{-\alpha} \\ k_t(i) &: r_t = \lambda^p \alpha Z_t k_t^p(i)^{\alpha-1} q_t(i)^{1-\alpha} n_t^p(i)^{1-\alpha} \end{aligned}$$

Divide

$$\begin{aligned} \frac{\tilde{w}_t^u}{r_t} &= \frac{1 - \alpha}{\alpha} \frac{k_t^p(i)}{n_t^p(i)} \\ k_t^p(i) &= \frac{\alpha}{1 - \alpha} \frac{\tilde{w}_t^u}{r_t} n_t^p(i) \\ n_t^p(i) &= \frac{1 - \alpha}{\alpha} \frac{r_t}{\tilde{w}_t^u} k_t^p(i) \end{aligned}$$

Production function

$$\begin{aligned} y_t(i) &= Z_t k_t^p(i)^\alpha [q_t(i) n_t^p(i)]^{1-\alpha} = Z_t k_t^p(i)^\alpha \left[q_t(i) \frac{1 - \alpha}{\alpha} \frac{r_t}{\tilde{w}_t^u} k_t^p(i) \right]^{1-\alpha} \\ &= Z_t k_t^p(i) \left[q_t(i) \frac{1 - \alpha}{\alpha} \frac{r_t}{\tilde{w}_t^u} \right]^{1-\alpha} \\ k_t^p(i) &= \frac{y_t(i)}{Z_t} \left[q_t(i) \frac{1 - \alpha}{\alpha} \frac{r_t}{\tilde{w}_t^u} \right]^{\alpha-1} \end{aligned}$$

Total cost

$$\begin{aligned} tc_t^p(i) &= \tilde{w}_t^u n_t^p(i) + r_t k_t^p(i) = \tilde{w}_t^u \frac{1 - \alpha}{\alpha} \frac{r_t}{\tilde{w}_t^u} k_t^p(i) + r_t k_t^p(i) = \left(\frac{1 - \alpha}{\alpha} + 1 \right) r_t k_t^p(i) = \frac{r_t}{\alpha} k_t^p(i) \\ &= \frac{r_t}{\alpha} \frac{y_t(i)}{Z_t} \left[q_t(i) \frac{1 - \alpha}{\alpha} \frac{r_t}{\tilde{w}_t^u} \right]^{\alpha-1} = \frac{y_t(i)}{Z_t} \left(\frac{r_t}{\alpha} \right)^\alpha \left(\frac{\tilde{w}_t^u / q_t(i)}{1 - \alpha} \right)^{1-\alpha} \end{aligned}$$

Real marginal cost

$$mc_t^p(i) = \frac{1}{Z_t} \left(\frac{r_t}{\alpha} \right)^\alpha \left(\frac{\tilde{w}_t^u / q_t(i)}{1 - \alpha} \right)^{1-\alpha}$$

Research and development sector

$$\begin{aligned} \min \quad & tc_t^x(i) = \tilde{w}_t^s n_t^x(i) + r_t k_t^x(i) \\ \text{subject to} \quad & x_t(i) = \frac{k_t^x(i)^\alpha [Q_t n_t^x(i)]^{1-\alpha}}{Q_t \phi_t(i)} \end{aligned}$$

FOCs

$$\begin{aligned} n_t^x(i) &: \tilde{w}_t^s = \lambda(1-\alpha) \frac{Z_t k_t^x(i)^\alpha Q_t^{1-\alpha} n_t^x(i)^{-\alpha}}{Q_t \phi_t(i)} \\ k_t^x(i) &: r_t = \lambda \alpha \frac{Z_t k_t^x(i)^{\alpha-1} Q_t^{1-\alpha} n_t^x(i)^{1-\alpha}}{Q_t \phi_t(i)} \end{aligned}$$

Divide

$$\begin{aligned} \frac{\tilde{w}_t^s}{r_t} &= \frac{1-\alpha}{\alpha} \frac{k_t^x(i)}{n_t^x(i)} \\ k_t^x(i) &= \frac{\alpha}{1-\alpha} \frac{\tilde{w}_t^s}{r_t} n_t^x(i) \\ n_t^x(i) &= \frac{1-\alpha}{\alpha} \frac{r_t}{\tilde{w}_t^s} k_t^x(i) \end{aligned}$$

R&D production function

$$\begin{aligned} x_t(i) &= \frac{k_t^x(i)^\alpha [Q_t n_t^x(i)]^{1-\alpha}}{Q_t \phi_t(i)} = Q_t^{-\alpha} k_t^x(i) \left(\frac{1-\alpha}{\alpha} \frac{r_t}{\tilde{w}_t^s} \right)^{1-\alpha} / \phi_t(i) \\ k_t^x(i) &= x_t(i) Q_t^\alpha \left(\frac{1-\alpha}{\alpha} \frac{r_t}{\tilde{w}_t^s} \right)^{\alpha-1} \phi_t(i) \end{aligned}$$

Total cost

$$\begin{aligned} tc_t^x(i) &= \frac{r_t}{\alpha} k_t^x(i) = \frac{r_t}{\alpha} x_t(i) Q_t^\alpha \left(\frac{1-\alpha}{\alpha} \frac{r_t}{\tilde{w}_t^s} \right)^{\alpha-1} \phi_t(i) \\ &= x_t(i) Q_t^\alpha \left(\frac{r_t}{\alpha} \right)^\alpha \left(\frac{\tilde{w}_t^s}{1-\alpha} \right)^{1-\alpha} \phi_t(i) \end{aligned}$$

Real marginal cost

$$mc_t^x(i) = Q_t^\alpha \left(\frac{r_t}{\alpha} \right)^\alpha \left(\frac{\tilde{w}_t^s}{1-\alpha} \right)^{1-\alpha} \phi_t(i) \equiv \bar{m} c_t^x \phi_t(i)$$

Total cost as function of desired innovative success probability

$$\begin{aligned} \chi_t(i) &= \frac{ax_t(i)}{1+ax_t(i)} \\ x_t(i) &= \frac{1}{a} \frac{\chi_t(i)}{1-\chi_t(i)} \\ tc_t^x(i) &= \frac{\bar{m} c_t^x}{a} \frac{\chi_t(i)}{1-\chi_t(i)} \phi_t(i) \end{aligned}$$

A.2.1.2 Aggregate production function

Relative inputs

$$\begin{aligned}
\frac{y_t(i)}{y_t(j)} &= \frac{Y_t p_t(i)^{-\sigma}}{Y_t p_t(j)^{-\sigma}} = \left[\frac{\frac{\sigma}{\sigma-1} \frac{1}{Z_t} \left(\frac{r_t}{\alpha}\right)^\alpha \left(\frac{\tilde{w}_t^u/q_t(i)}{1-\alpha}\right)^{1-\alpha}}{\frac{\sigma}{\sigma-1} \frac{1}{Z_t} \left(\frac{r_t}{\alpha}\right)^\alpha \left(\frac{\tilde{w}_t^u/q_t(j)}{1-\alpha}\right)^{1-\alpha}} \right]^{-\sigma} = \left(\frac{q_t(i)^{\alpha-1}}{q_t(j)^{\alpha-1}} \right)^{-\sigma} = \left(\frac{q_t(i)^{1-\alpha}}{q_t(j)^{1-\alpha}} \right)^\sigma \\
\frac{y_t(i)}{y_t(j)} &= \frac{Z_t k_t^p(i) \left[q_t(i) \frac{1-\alpha}{\alpha} \frac{r_t}{\tilde{w}_t^u} \right]^{1-\alpha}}{Z_t k_t^p(j) \left[q_t(j) \frac{1-\alpha}{\alpha} \frac{r_t}{\tilde{w}_t^u} \right]^{1-\alpha}} \\
\frac{k_t^p(i) q_t(i)^{1-\alpha}}{k_t^p(j) q_t(j)^{1-\alpha}} &= \left(\frac{q_t(i)^{1-\alpha}}{q_t(j)^{1-\alpha}} \right)^\sigma \\
\frac{k_t^p(i)}{k_t^p(j)} &= \left(\frac{q_t(i)}{q_t(j)} \right)^{(1-\alpha)(\sigma-1)} \\
k_t^p(i) &= \left(\frac{q_t(i)}{q_t(j)} \right)^{(1-\alpha)(\sigma-1)} k_t^p(j) \\
k_t^p(i) &= \left(\frac{q_t(i)}{Q_t} \right)^{(1-\alpha)(\sigma-1)} \bar{k}_t^p \\
n_t^p(i) &= \left(\frac{q_t(i)}{Q_t} \right)^{(1-\alpha)(\sigma-1)} \bar{n}_t^p
\end{aligned}$$

where $\bar{k}_t^p \equiv K_t^p/M_t$ and $\bar{n}_t^p \equiv N_t^p/M_t$.

Final goods output

$$\begin{aligned}
Y_t &= \left[\int_0^{M_t} y_t(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}} = \left[M_t \int_0^\infty y_t(q)^{\frac{\sigma-1}{\sigma}} \mu_t(q) dq \right]^{\frac{\sigma}{\sigma-1}} \\
&= M_t^{\frac{\sigma}{\sigma-1}} \left[\int_0^\infty \left[Z_t k_t^p(q)^\alpha q^{1-\alpha} n_t^p(q)^{1-\alpha} \right]^{\frac{\sigma-1}{\sigma}} \mu_t(q) dq \right]^{\frac{\sigma}{\sigma-1}} \\
&= M_t^{\frac{\sigma}{\sigma-1}} Z_t \left[\int_0^\infty \left[\left(\frac{q}{Q_t} \right)^{(1-\alpha)(\sigma-1)} (\bar{k}_t^p)^\alpha (\bar{n}_t^p)^{1-\alpha} q^{1-\alpha} \right]^{\frac{\sigma-1}{\sigma}} \mu_t(q) dq \right]^{\frac{\sigma}{\sigma-1}} \\
&= M_t^{\frac{\sigma}{\sigma-1}} Z_t (\bar{k}_t^p)^\alpha (\bar{n}_t^p)^{1-\alpha} Q_t^{(1-\alpha)(1-\sigma)} \left[\int_0^\infty \left[(q^{1-\alpha})^\sigma \right]^{\frac{\sigma-1}{\sigma}} \mu_t(q) dq \right]^{\frac{\sigma}{\sigma-1}} \\
&= M_t^{\frac{\sigma}{\sigma-1}-1} Z_t (K_t^p)^\alpha (N_t^p)^{1-\alpha} Q_t^{(1-\alpha)(1-\sigma)} \left[\int_0^\infty (q^{1-\alpha})^{\sigma-1} \mu_t(q) dq \right]^{\frac{1}{\sigma-1}}^\sigma \\
&= M_t^{\frac{1}{\sigma-1}} Z_t (K_t^p)^\alpha (N_t^p)^{1-\alpha} Q_t^{(1-\alpha)(1-\sigma)} (Q_t^{1-\alpha})^\sigma \\
&= M_t^{\frac{1}{\sigma-1}} Z_t (K_t^p)^\alpha (Q_t N_t^p)^{1-\alpha}
\end{aligned}$$

A.2.1.3 Real profit function

Real operating profit

$$\begin{aligned}
\pi_t^o(i) &= p_t(i) y_t(i) - m c_t^p(i) y_t(i) - f_t = p_t(i) y_t(i) - p_t(i) \frac{\sigma-1}{\sigma} y_t(i) - f_t \\
&= \left(1 - \frac{\sigma-1}{\sigma}\right) Y_t p_t(i)^{1-\sigma} - f_t = \frac{1}{\sigma} Y_t \left[\frac{\sigma}{\sigma-1} \frac{1}{Z_t} \left(\frac{r_t}{\alpha}\right)^\alpha \left(\frac{\tilde{w}_t^u/q_t(i)}{1-\alpha}\right)^{1-\alpha} \right]^{1-\sigma} - f_t \\
&= \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} Y_t Z_t^{\sigma-1} \left[\left(\frac{r_t}{\alpha}\right)^\alpha \left(\frac{\tilde{w}_t^u/q_t(i)}{1-\alpha}\right)^{1-\alpha} \right]^{1-\sigma} - f_t
\end{aligned}$$

Price index (where $R_t \equiv P_t r_t$ and $W_t^u \equiv P_t \tilde{w}_t^u$)

$$\begin{aligned}
P_t &= \left[\int_0^{M_t} P_t(i)^{1-\sigma} di \right]^{\frac{1}{1-\sigma}} = \left[M_t \int_0^\infty P_t(q)^{1-\sigma} \mu_t(q) dq \right]^{\frac{1}{1-\sigma}} \\
&= M_t^{\frac{1}{1-\sigma}} \left[\int_0^\infty \left[\frac{\sigma}{\sigma-1} \frac{1}{Z_t} \left(\frac{R_t}{\alpha}\right)^\alpha \left(\frac{W_t^u/q}{1-\alpha}\right)^{1-\alpha} \right]^{1-\sigma} \mu_t(q) dq \right]^{\frac{1}{1-\sigma}} \\
&= \frac{\sigma}{\sigma-1} M_t^{\frac{1}{1-\sigma}} \frac{1}{Z_t} \left(\frac{R_t}{\alpha}\right)^\alpha \left(\frac{W_t^u}{1-\alpha}\right)^{1-\alpha} \left[\int_0^\infty (q^{\alpha-1})^{1-\sigma} \mu_t(q) dq \right]^{\frac{1}{1-\sigma}} \\
&= \frac{\sigma}{\sigma-1} M_t^{\frac{1}{1-\sigma}} \frac{1}{Z_t} \left(\frac{R_t}{\alpha}\right)^\alpha \left(\frac{W_t^u}{1-\alpha}\right)^{1-\alpha} \left[\left[\int_0^\infty (q^{1-\alpha})^{\sigma-1} \mu_t(q) dq \right]^{\frac{1}{\sigma-1}} \right]^{-1} \\
&= \frac{\sigma}{\sigma-1} M_t^{\frac{1}{1-\sigma}} \frac{1}{Z_t} \left(\frac{R_t}{\alpha}\right)^\alpha \left(\frac{W_t^u}{1-\alpha}\right)^{1-\alpha} (Q_t^{1-\alpha})^{-1} \\
&= \frac{\sigma}{\sigma-1} M_t^{\frac{1}{1-\sigma}} \frac{1}{Z_t} \left(\frac{R_t}{\alpha}\right)^\alpha \left(\frac{W_t^u/Q_t}{1-\alpha}\right)^{1-\alpha}
\end{aligned}$$

Real input cost index

$$\begin{aligned}
\left(\frac{R_t}{\alpha}\right)^\alpha \left(\frac{W_t^u/Q_t}{1-\alpha}\right)^{1-\alpha} &= \frac{\sigma-1}{\sigma} P_t M_t^{\frac{1}{\sigma-1}} Z_t \\
\left(\frac{r_t}{\alpha}\right)^\alpha \left(\frac{\tilde{w}_t^u}{1-\alpha}\right)^{1-\alpha} &= \frac{\sigma-1}{\sigma} M_t^{\frac{1}{\sigma-1}} Z_t Q_t^{1-\alpha} \\
\frac{1}{Z_t} \left(\frac{r_t}{\alpha}\right)^\alpha \left(\frac{\tilde{w}_t^u}{1-\alpha}\right)^{1-\alpha} &= \frac{\sigma-1}{\sigma} M_t^{\frac{1}{\sigma-1}} Q_t^{1-\alpha}
\end{aligned}$$

Real operating profit

$$\begin{aligned}
\pi_t^o(i) &= \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} Y_t Z_t^{\sigma-1} \left[\left(\frac{r_t}{\alpha} \right)^\alpha \left(\frac{w_t/q_t(i)}{1-\alpha} \right)^{1-\alpha} \right]^{1-\sigma} - f_t \\
&= \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} Y_t Z_t^{\sigma-1} \left[\frac{\sigma-1}{\sigma} M_t^{\frac{1}{\sigma-1}} Z_t Q_t^{1-\alpha} q_t(i)^{\alpha-1} \right]^{1-\sigma} - f_t \\
&= \frac{Y_t}{\sigma M_t} \left[\left(\frac{q_t(i)}{Q_t} \right)^{1-\alpha} \right]^{\sigma-1} - f_t \\
&= \frac{Y_t}{\sigma M_t} \phi_t(i) - f_t
\end{aligned}$$

Real profit

$$\begin{aligned}
\pi_t(i) &= \pi_t^o(i) - \frac{\bar{m}c_t^x}{a} \frac{\chi_t(i)}{1-\chi_t(i)} \phi_t(i) \\
&= \left(\frac{Y_t}{\sigma M_t} - \frac{\bar{m}c_t^x}{a} \frac{\chi_t(i)}{1-\chi_t(i)} \right) \phi_t(i) - f_t \\
&= \left(\frac{Y_t}{\sigma M_t} - \frac{\bar{m}c_t^x}{a} \frac{\chi_t(i)}{1-\chi_t(i)} \right) \phi_t(i) - \bar{m}c_t^x f \\
&= Y_t \left[\left(\frac{1}{\sigma M_t} - \frac{\omega_t}{a} \frac{\chi_t(i)}{1-\chi_t(i)} \right) \phi_t(i) - \omega_t f \right]
\end{aligned}$$

A.2.1.4 Evolution of aggregate quality index

Following Melitz (2003), I consider the current period distribution of quality levels $\mu_t(q)$ to be a truncated part of an underlying distribution $g_t(q)$, so that:

$$\mu_t(q) = \begin{cases} 1/[1 - G_t(q_{t-1}^*)] g_t(q) & \text{if } q \geq q_{t-1}^* \\ 0 & \text{otherwise} \end{cases}$$

where $q_t^* = (\phi_t^*)^{1/[(1-\alpha)(\sigma-1)]} Q_t$.

Aggregate quality index at the end of period t :

$$Q_t^{1-\alpha} = \left[\int_0^\infty (q^{1-\alpha})^{\sigma-1} \mu_t(q) dq \right]^{\frac{1}{\sigma-1}} = \left[\frac{1}{1 - G_t(q_{t-1}^*)} \int_{q_{t-1}^*}^\infty (q^{1-\alpha})^{\sigma-1} g_t(q) dq \right]^{\frac{1}{\sigma-1}}$$

The aggregate quality level after exits and innovation resolution but before entry:

$$\begin{aligned}
Q_t^* &= \left\{ \frac{1}{1 - G_t(q_t^*)} \left[(1 - \chi_t) \int_{q_t^*}^\infty (q^{1-\alpha})^{\sigma-1} g_t(q) dq + \chi_t \int_{q_t^*}^\infty \left(\iota^{\frac{1}{(1-\alpha)(\sigma-1)}} q \right)^{(1-\alpha)(\sigma-1)} g_t(q) dq \right] \right\}^{\frac{1}{\sigma-1}} \\
&= \left[(1 - \chi_t + \chi_t \iota) \frac{1}{1 - G_t(q_t^*)} \int_{q_t^*}^\infty (q^{1-\alpha})^{\sigma-1} g_t(q) dq \right]^{\frac{1}{\sigma-1}}
\end{aligned}$$

Aggregate quality index in $t + 1$ after entry:

$$\begin{aligned}
Q_{t+1} &= \left\{ \frac{1 - \chi_t + \chi_t \ell}{1 - G_t(q_t^*)} \left[\frac{\left(1 - \frac{M_t^e}{M_{t+1}}\right) \int_{q_t^*}^{\infty} (q^{1-\alpha})^{\sigma-1} g_t(q) dq}{+ \frac{M_t^e}{M_{t+1}} \int_{q_t^*}^{\infty} \left(\left(\frac{\sigma}{\sigma-1}\right)^{\frac{1}{(1-\alpha)(\sigma-1)}} q\right)^{(1-\alpha)(\sigma-1)} g_t(q) dq} \right] \right\}^{\frac{1}{\sigma-1}} \\
&= \left[(1 - \chi_t + \chi_t \ell) \left(1 - \frac{M_t^e}{M_{t+1}} + \frac{M_t^e}{M_{t+1}} \frac{\sigma}{\sigma-1}\right) \frac{1}{1 - G_t(q_t^*)} \int_{q_t^*}^{\infty} (q^{1-\alpha})^{\sigma-1} g_t(q) dq \right]^{\frac{1}{\sigma-1}}
\end{aligned}$$

Transformed aggregate growth rate η_t :

$$\begin{aligned}
\eta_t &= \left(\frac{Q_{t+1}}{Q_t} \right)^{(1-\alpha)(\sigma-1)} \\
&= \left\{ \frac{\left[(1 - \chi_t + \chi_t \ell) \left(1 - \frac{M_t^e}{M_{t+1}} + \frac{M_t^e}{M_{t+1}} \frac{\sigma}{\sigma-1}\right) \frac{1}{1 - G_t(q_t^*)} \int_{q_t^*}^{\infty} (q^{1-\alpha})^{\sigma-1} g_t(q) dq \right]^{\frac{1}{\sigma-1}}}{\left[\frac{1}{1 - G_t(q_{t-1}^*)} \int_{q_{t-1}^*}^{\infty} (q^{1-\alpha})^{\sigma-1} g_t(q) dq \right]^{\frac{1}{\sigma-1}}} } \right\}^{\sigma-1} \\
&\approx (1 - \chi_t + \chi_t \ell) \left(1 - \frac{M_t^e}{M_{t+1}} + \frac{M_t^e}{M_{t+1}} \frac{\sigma}{\sigma-1}\right)
\end{aligned}$$

where if the distribution is invariant with respect to the cutoff points q_{t-1}^* and q_t^* (as is the case with Pareto and other power-law distributions) then the above relationship holds with equality.

A.2.1.5 Target wage

Expression for target wage

$$w_t^o = w_t^f + \psi \left(\frac{\kappa}{2} (x_t^2(r) - x_t^2) - p_t \kappa x_t \right) + (1 - \psi) p_t E_t [\beta \Lambda_{t,t+1} H_{t+1}]$$

Average vs conditional on renegotiation worker surplus

$$H_t = H_t(r) + \Delta_t(w_t - w_t(r))$$

Therefore

$$\begin{aligned}
(1 - \psi) p_t E_t [\beta \Lambda_{t,t+1} H_{t+1}] &= \\
&= (1 - \psi) p_t E_t [\beta \Lambda_{t,t+1} [H_{t+1}(r) + \lambda \Delta_{t+1}(w_{t+1} - w_{t+1}(r))]] \\
&= (1 - \psi) p_t E_t [\beta \Lambda_{t,t+1} H_{t+1}(r)] + (1 - \psi) p_t E_t [\beta \Lambda_{t,t+1} \lambda \Delta_{t+1}(w_{t+1} - w_{t+1}(r))] \\
&= \psi p_t E_t [\beta \Lambda_{t,t+1} J_{t+1}(r)] + (1 - \psi) p_t E_t [\beta \Lambda_{t,t+1} \lambda \Delta_{t+1}(w_{t+1} - w_{t+1}(r))] \\
&= \psi p_t \kappa x_t(r) + (1 - \psi) p_t E_t [\beta \Lambda_{t,t+1} \lambda \Delta_{t+1}(w_{t+1} - w_{t+1}(r))]
\end{aligned}$$

Resulting target wage

$$\begin{aligned}
w_t^o &= w_t^f + \psi \left(\frac{\kappa}{2} (x_t^2(r) - x_t^2) + p_t \kappa (x_t(r) - x_t) \right) \\
&\quad + (1 - \psi) p_t E_t [\beta \Lambda_{t,t+1} \lambda \Delta_{t+1}(w_{t+1} - w_{t+1}(r))]
\end{aligned}$$

A.2.2 Full set of model equations

Stationarized variables notation

$$\hat{X}_t \equiv X_t/Q_t$$

Stationarizing variables

$$g_t^Q \equiv Q_{t+1}/Q_t = \eta_t^{\frac{1}{(1-\alpha)(\sigma-1)}} \quad (\text{A.2.1})$$

$$\gamma_{t,t+1} \equiv Y_{t+1}/Y_t = g_t^Q \cdot \hat{Y}_{t+1}/\hat{Y}_t \quad (\text{A.2.2})$$

Incumbents' problem

$$\phi_t = 1 \quad (\text{A.2.3})$$

$$v_t = A_t + B_t \phi_t \quad (\text{A.2.4})$$

$$\pi_t = \left(\frac{1}{\sigma M_t} - \frac{\omega_t}{a} \frac{\chi_t}{1 - \chi_t} \right) \phi_t - \omega_t f \quad (\text{A.2.5})$$

$$A_t + B_t \phi_t = \pi_t + \text{E}_t \left[\beta \Lambda_{t,t+1} (1 - \delta_t) \gamma_{t,t+1} \left(A_{t+1} + B_{t+1} \frac{\chi_t (\iota - 1) + 1}{\eta_t} \phi_t \right) \right] \quad (\text{A.2.6})$$

$$0 = -\frac{\omega_t}{a} \frac{1}{(1 - \chi_t)^2} + \text{E}_t \left[\beta \Lambda_{t,t+1} (1 - \delta_t) \gamma_{t,t+1} B_{t+1} \frac{(\iota - 1) \phi_t}{\eta_t} \right] \quad (\text{A.2.7})$$

$$B_t = \frac{1}{\sigma M_t} - \frac{\omega_t}{a} \frac{\chi_t}{1 - \chi_t} + \text{E}_t \left[\beta \Lambda_{t,t+1} (1 - \delta_t) \gamma_{t,t+1} B_{t+1} \frac{\chi_t (\iota - 1) + 1}{\eta_t} \right] \quad (\text{A.2.8})$$

Entrants' problem

$$v_t^e = -\omega_t \left(f^e + \frac{1}{a^e} \frac{\chi_t^e}{1 - \chi_t^e} \right) + \chi_t^e \text{E}_t \left[\beta \Lambda_{t,t+1} \gamma_{t,t+1} \left(A_{t+1} + B_{t+1} \frac{\sigma}{\sigma - 1} \phi_{t+1} \right) \right] \quad (\text{A.2.9})$$

$$0 = -\frac{\omega_t}{a^e} \frac{1}{(1 - \chi_t^e)^2} + \text{E}_t \left[\beta \Lambda_{t,t+1} \gamma_{t,t+1} \left(A_{t+1} + B_{t+1} \frac{\sigma}{\sigma - 1} \phi_{t+1} \right) \right] \quad (\text{A.2.10})$$

$$v_t^e = 0 \quad (\text{A.2.11})$$

Establishment dynamics

$$\delta_t = 1 - (1 - \delta^{exo}) (1 - M_t^e) \quad (\text{A.2.12})$$

$$\frac{\omega_t}{a} \frac{\chi_t}{1 - \chi_t} \phi_t^* = \text{E}_t \left[\beta \Lambda_{t,t+1} (1 - \delta_t) \gamma_{t,t+1} \left(A_{t+1} + B_{t+1} \frac{\chi_t (\iota - 1) + 1}{\eta_t} \phi_t^* \right) \right] \quad (\text{A.2.13})$$

$$M_t^x = M_t (1 - \chi_{t-1}) \left(1 - \frac{\phi_{t-1}^*}{\phi_t^* \eta_{t-1}} \right) \quad (\text{A.2.14})$$

$$M_{t+1} = (1 - \delta_t) (M_t - M_t^x) + M_t^e \quad (\text{A.2.15})$$

$$\eta_t = (1 - \chi_t + \chi_t \iota) \left(1 - \frac{M_t^e}{M_{t+1}} + \frac{M_t^e}{M_{t+1}} \frac{\sigma}{\sigma - 1} \right) \quad (\text{A.2.16})$$

Skilled sector

$$\omega_t \hat{Y}_t = \left(\frac{r_t^k}{\alpha} \right)^\alpha \left(\frac{\hat{w}_t^s}{1 - \alpha} \right)^{1 - \alpha} \quad (\text{A.2.17})$$

$$(\hat{K}_t^s)^\alpha (N_t^s)^{1 - \alpha} = M_t f + (M_t - M_t^x) \left(\frac{1}{a} \frac{\chi_t}{1 - \chi_t} \right) + \frac{M_t^e}{\chi_t^e} \left(f^e + \frac{1}{a^e} \frac{\chi_t^e}{1 - \chi_t^e} \right) \quad (\text{A.2.18})$$

$$\frac{r_t^k}{\hat{w}_t^s} = \frac{\alpha}{1 - \alpha} \frac{N_t^s}{\hat{K}_t^s} \quad (\text{A.2.19})$$

Unskilled sector

$$\hat{Y}_t = Z_t M_t^{\frac{1}{\sigma - 1}} (\hat{K}_t^p)^\alpha (N_t^p)^{1 - \alpha} \quad (\text{A.2.20})$$

$$\hat{w}_t^u = (1 - \alpha) \frac{\sigma - 1}{\sigma} Z_t M_t^{\frac{1}{\sigma - 1}} (\hat{K}_t^p)^\alpha (N_t^p)^{-\alpha} \quad (\text{A.2.21})$$

$$r_t^k = \alpha \frac{\sigma - 1}{\sigma} Z_t M_t^{\frac{1}{\sigma - 1}} (\hat{K}_t^p)^{\alpha - 1} (N_t^p)^{1 - \alpha} \quad (\text{A.2.22})$$

Households

$$1 = \text{E}_t \left[\beta \left(g_t^Q \cdot \hat{C}_{t+1} / \hat{C}_t \right)^{-\theta} (1 + r_t^k - dp) \right] \quad (\text{A.2.23})$$

$$\Lambda_{t,t+1} = \text{E}_t \left[\left(g_t^Q \cdot \hat{C}_{t+1} / \hat{C}_t \right)^{-\theta} \right] \quad (\text{A.2.24})$$

Frictional labor markets (notation $w_t^* \equiv w_t(r)$)

$$m_t^u = \sigma_m (u_t^u)^\psi (v_t^u)^{1 - \psi} \quad (\text{A.2.25})$$

$$m_t^s = \sigma_m (u_t^s)^\psi (v_t^s)^{1 - \psi} \quad (\text{A.2.26})$$

$$n_{t+1}^u = (\rho^u + x_t^u) n_t^u \quad (\text{A.2.27})$$

$$n_{t+1}^s = (\rho^s + x_t^s) n_t^s \quad (\text{A.2.28})$$

$$u_t^u = 1 - n_t^u \quad (\text{A.2.29})$$

$$u_t^s = 1 - n_t^s \quad (\text{A.2.30})$$

$$q_t^u = m_t^u / v_t^u \quad (\text{A.2.31})$$

$$q_t^s = m_t^s / v_t^s \quad (\text{A.2.32})$$

$$p_t^u = m_t^u / u_t^u \quad (\text{A.2.33})$$

$$p_t^s = m_t^s / u_t^s \quad (\text{A.2.34})$$

$$x_t^u = q_t^u v_t^u / n_t^u \quad (\text{A.2.35})$$

$$x_t^s = q_t^s v_t^s / n_t^s \quad (\text{A.2.36})$$

$$\kappa^u x_t^u = E_t \left[\beta \Lambda_{t,t+1} \left(\hat{w}_{t+1}^u - \hat{w}_t^u + \frac{\kappa^u}{2} (x_{t+1}^u)^2 + \rho^u \kappa^u x_{t+1}^u \right) \right] \quad (\text{A.2.37})$$

$$\kappa^s x_t^s = E_t \left[\beta \Lambda_{t,t+1} \left(\hat{w}_{t+1}^s - \hat{w}_t^s + \frac{\kappa^s}{2} (x_{t+1}^s)^2 + \rho^s \kappa^s x_{t+1}^s \right) \right] \quad (\text{A.2.38})$$

$$\kappa^u x_t^{u*} = E_t \left[\beta \Lambda_{t,t+1} \left(\hat{w}_{t+1}^u - \hat{w}_t^{u*} + \frac{\kappa^u}{2} (x_{t+1}^{u*})^2 + \rho^u \kappa^u x_{t+1}^{u*} \right) \right] \quad (\text{A.2.39})$$

$$\kappa^s x_t^{s*} = E_t \left[\beta \Lambda_{t,t+1} \left(\hat{w}_{t+1}^s - \hat{w}_t^{s*} + \frac{\kappa^s}{2} (x_{t+1}^{s*})^2 + \rho^s \kappa^s x_{t+1}^{s*} \right) \right] \quad (\text{A.2.40})$$

$$\Delta_t^u = 1 + \beta \rho^u \lambda E_t \left[\Lambda_{t,t+1} g_t^Q \Delta_{t+1}^u \right] \quad (\text{A.2.41})$$

$$\Delta_t^s = 1 + \beta \rho^s \lambda E_t \left[\Lambda_{t,t+1} g_t^Q \Delta_{t+1}^s \right] \quad (\text{A.2.42})$$

$$\Delta_t^u \hat{w}_t^{u*} = \hat{w}_t^{uo} + \rho^u \lambda E_t \left[\beta \Lambda_{t,t+1} \Delta_{t+1}^u \hat{w}_{t+1}^{u*} \right] \quad (\text{A.2.43})$$

$$\Delta_t^s \hat{w}_t^{s*} = \hat{w}_t^{so} + \rho^s \lambda E_t \left[\beta \Lambda_{t,t+1} \Delta_{t+1}^s \hat{w}_{t+1}^{s*} \right] \quad (\text{A.2.44})$$

$$\hat{w}_t^{uf} = \psi \left(\hat{w}_t^u + \frac{\kappa^u}{2} (x_t^u)^2 + p_t^u \kappa^u x_t^u \right) + (1 - \psi) b_t^u \quad (\text{A.2.45})$$

$$\hat{w}_t^{sf} = \psi \left(\hat{w}_t^s + \frac{\kappa^s}{2} (x_t^s)^2 + p_t^s \kappa^s x_t^s \right) + (1 - \psi) b_t^s \quad (\text{A.2.46})$$

$$\begin{aligned} \hat{w}_t^{uo} &= \hat{w}_t^{uf} + \psi \left(\frac{\kappa^u}{2} ((x_t^{u*})^2 - (x_t^u)^2) + p_t^u \kappa^u (x_t^{u*} - x_t^u) \right) \\ &\quad + (1 - \psi) p_t^u E_t \left[\beta \Lambda_{t,t+1} \lambda \Delta_{t+1}^u g_t^Q (\hat{w}_t^u - \hat{w}_t^{u*}) \right] \end{aligned} \quad (\text{A.2.47})$$

$$\begin{aligned} \hat{w}_t^{so} &= \hat{w}_t^{sf} + \psi \left(\frac{\kappa^s}{2} ((x_t^{s*})^2 - (x_t^s)^2) + p_t^s \kappa^s (x_t^{s*} - x_t^s) \right) \\ &\quad + (1 - \psi) p_t^s E_t \left[\beta \Lambda_{t,t+1} \lambda \Delta_{t+1}^s g_t^Q (\hat{w}_t^s - \hat{w}_t^{s*}) \right] \end{aligned} \quad (\text{A.2.48})$$

$$\hat{w}_t^u = \lambda \hat{w}_{t-1}^u + (1 - \lambda) \hat{w}_t^{u*} \quad (\text{A.2.49})$$

$$\hat{w}_t^s = \lambda \hat{w}_{t-1}^s + (1 - \lambda) \hat{w}_t^{s*} \quad (\text{A.2.50})$$

$$\hat{b}_t^u = 0.4 \hat{w}_{ss}^u \quad (\text{A.2.51})$$

$$\hat{b}_t^s = 0.4 \hat{w}_{ss}^s \quad (\text{A.2.52})$$

Market clearing

$$\hat{Y}_t = \hat{C}_t + \hat{I}_t + \kappa^u (x_t^u)^2 N_t^p + \kappa^s (x_t^s)^2 N_t^s \quad (\text{A.2.53})$$

$$g_t^Q \hat{K}_{t+1} = (1 - dp) \hat{K}_t + \hat{I}_t \quad (\text{A.2.54})$$

$$\hat{K}_t = \hat{K}_t^p + \hat{K}_t^s \quad (\text{A.2.55})$$

$$N_t^p = (1 - s) n_t^u \quad (\text{A.2.56})$$

$$N_t^s = s n_t^s \quad (\text{A.2.57})$$

Shock

$$\log Z_t = \rho_Z \log Z_{t-1} + \varepsilon_{Z,t} \quad (\text{A.2.58})$$

Welfare

$$U_t = \frac{(\hat{C}_t Q_t)^{1-\theta}}{1-\theta} + \beta E_t[U_{t+1}] \quad (\text{A.2.59})$$

A.2.3 Additional tables and figures

Table A.1: Chapter 3: Prior distributions of parameters

Parameter	Description	Distribution shape	Mean	Std. dev.
λ	Average contract duration	Uniform $[0, 1]$	0.5	0.289
ρ_Z	Autocorr. of TFP process	Beta	0.7	0.175
σ_Z	Std. dev. of TFP shock	Inverse Gamma	0.01	∞

Figure A.1: Chapter 3: Results of the maximum likelihood estimation step

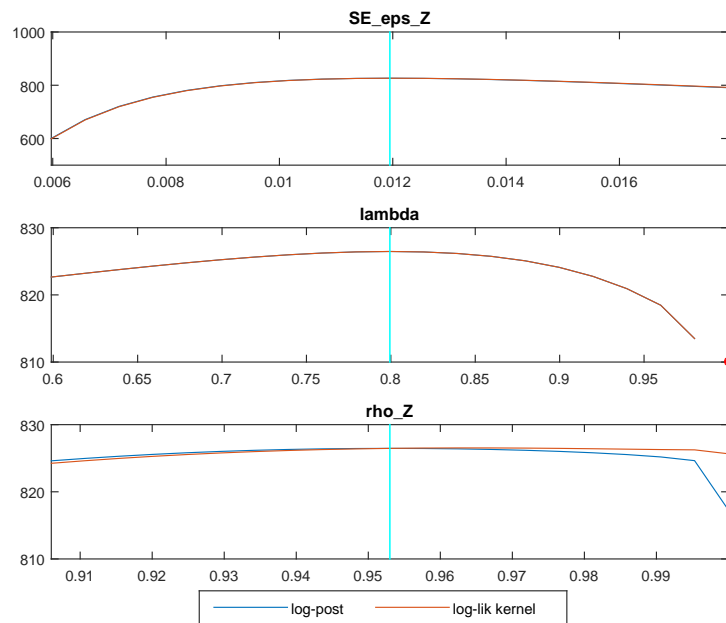


Figure A.2: Chapter 3: Univariate convergence of parameters

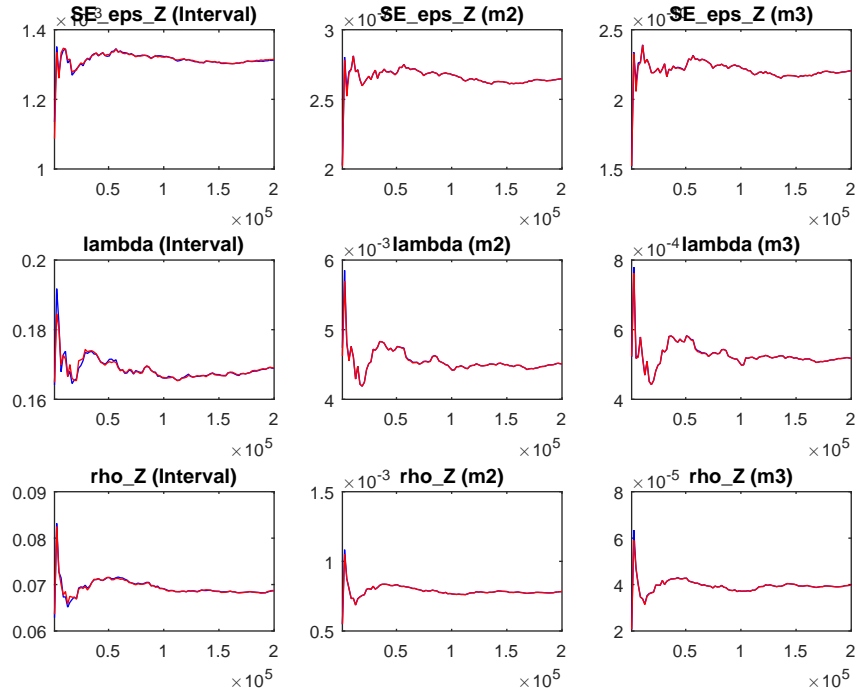


Figure A.3: Chapter 3: Multivariate convergence of parameters

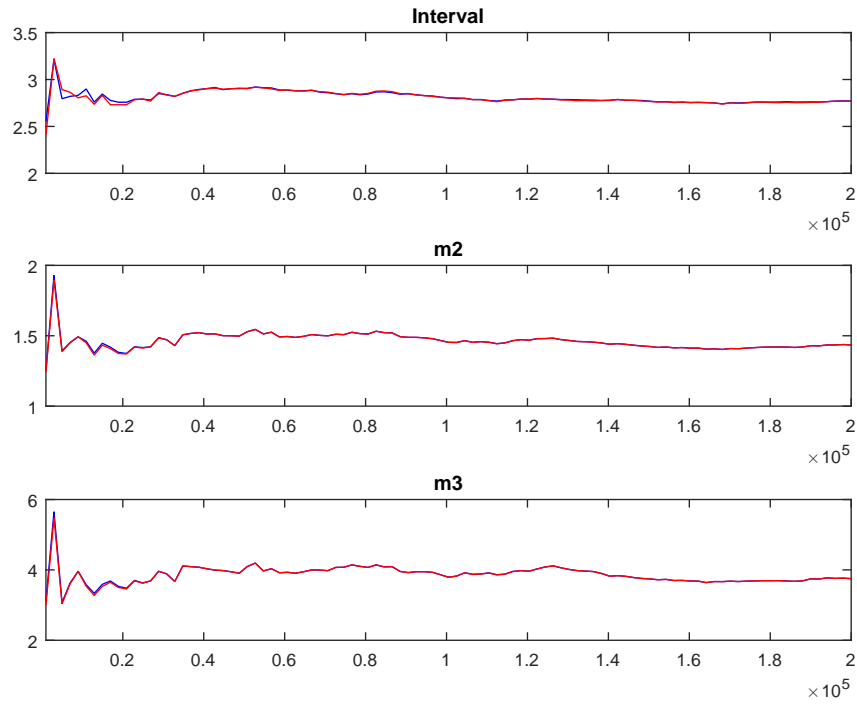
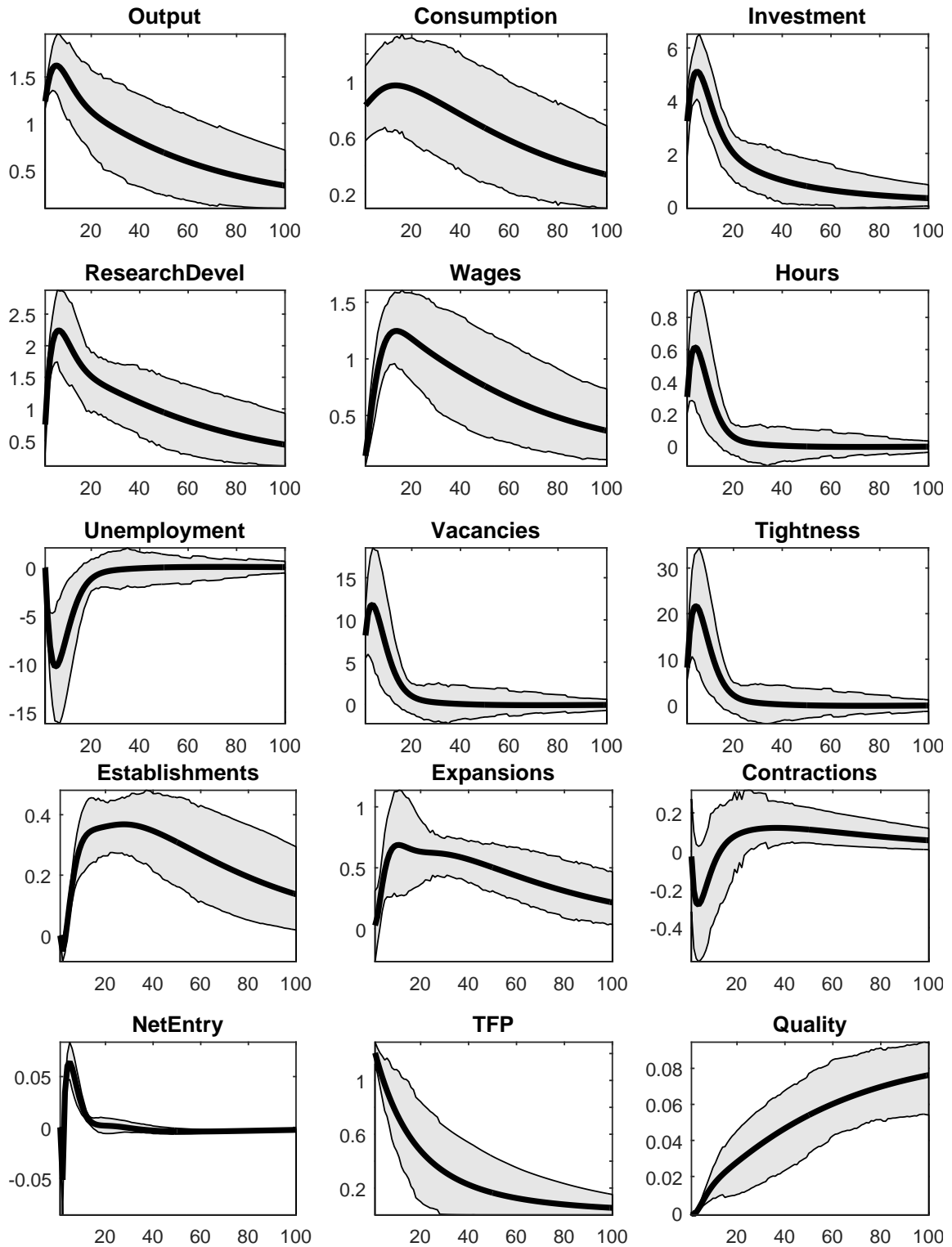


Figure A.4: Chapter 3: Bayesian impulse response functions



A.3 Chapter 4

A.3.1 Full set of stationarized model equations

Stationarized variables notation

$$\hat{X}_t \equiv X_t/Q_t$$

Stationarizing variables

$$g_t^Q \equiv Q_{t+1}/Q_t = \eta_t^{\frac{1}{(1-\alpha)(\sigma-1)}} \quad (\text{A.3.1})$$

$$\gamma_{t,t+1} \equiv Y_{t+1}/Y_t = g_t^Q \cdot \hat{Y}_{t+1}/\hat{Y}_t \quad (\text{A.3.2})$$

Incumbents' problem

$$\phi_t = 1 \quad (\text{A.3.3})$$

$$v_t = A_t + B_t \phi_t \quad (\text{A.3.4})$$

$$\pi_t = \left(\frac{1}{\sigma M_t} - \left(1 + \zeta r_t^l\right) \frac{\omega_t}{a} \frac{\chi_t}{1 - \chi_t} \right) \phi_t - \left(1 + \zeta r_t^l\right) \omega_t f \quad (\text{A.3.5})$$

$$A_t + B_t \phi_t = \pi_t + \text{E}_t \left[\Lambda_{t,t+1} (1 - \delta_t) \gamma_{t,t+1} \left(A_{t+1} + B_{t+1} \frac{\chi_t (\iota - 1) + 1}{\eta_t} \phi_t \right) \right] \quad (\text{A.3.6})$$

$$0 = - \left(1 + \zeta r_t^l\right) \frac{\omega_t}{a} \frac{1}{(1 - \chi_t)^2} + \text{E}_t \left[\beta \Lambda_{t,t+1} (1 - \delta_t) \gamma_{t,t+1} B_{t+1} \frac{(\iota - 1) \phi_t}{\eta_t} \right] \quad (\text{A.3.7})$$

$$B_t = \left(\frac{1}{\sigma M_t} - \left(1 + \zeta r_t^l\right) \frac{\omega_t}{a} \frac{\chi_t}{1 - \chi_t} \right) + \text{E}_t \left[\beta \Lambda_{t,t+1} (1 - \delta_t) \gamma_{t,t+1} B_{t+1} \frac{\chi_t (\iota - 1) + 1}{\eta_t} \right] \quad (\text{A.3.8})$$

Entrants' problem

$$v_t^e = - \left(1 + \zeta^e r_t^l\right) \omega_t \left(f^e + \frac{1}{a^e} \frac{\chi_t^e}{1 - \chi_t^e} \right) + \chi_t^e \text{E}_t \left[\beta \Lambda_{t,t+1} \gamma_{t,t+1} \left(A_{t+1} + B_{t+1} \frac{\sigma}{\sigma - 1} \phi_{t+1} \right) \right] \quad (\text{A.3.9})$$

$$0 = - \left(1 + \zeta^e r_t^l\right) \frac{\omega_t}{a^e} \frac{1}{(1 - \chi_t^e)^2} + \text{E}_t \left[\beta \Lambda_{t,t+1} \gamma_{t,t+1} \left(A_{t+1} + B_{t+1} \frac{\sigma}{\sigma - 1} \phi_{t+1} \right) \right] \quad (\text{A.3.10})$$

$$v_t^e = 0 \quad (\text{A.3.11})$$

Establishment dynamics

$$\delta_t = 1 - (1 - \delta^{exo}) (1 - M_t^e) \quad (\text{A.3.12})$$

$$\left(1 + \zeta r_t^l\right) \frac{\omega_t}{a} \frac{\chi_t}{1 - \chi_t} \phi_t^* = \text{E}_t \left[\beta \Lambda_{t,t+1} (1 - \delta_t) \gamma_{t,t+1} \left(A_{t+1} + B_{t+1} \frac{\chi_t (\iota - 1) + 1}{\eta_t} \phi_t^* \right) \right] \quad (\text{A.3.13})$$

$$M_t^x = M_t (1 - \chi_{t-1}) \left(1 - \frac{\phi_{t-1}^*}{\phi_t^* \eta_{t-1}} \right) \quad (\text{A.3.14})$$

$$M_{t+1} = (1 - \delta_t) (M_t - M_t^x) + M_t^e \quad (\text{A.3.15})$$

$$\eta_t = (1 - \chi_t + \chi_t \iota) \left(1 - \frac{M_t^e}{M_{t+1}} + \frac{M_t^e}{M_{t+1}} \frac{\sigma}{\sigma - 1} \right) \quad (\text{A.3.16})$$

Skilled sector

$$\omega_t \hat{Y}_t = \left(\frac{r_t^k}{\alpha} \right)^\alpha \left(\frac{\hat{w}_t^s}{1 - \alpha} \right)^{1 - \alpha} \quad (\text{A.3.17})$$

$$(\hat{K}_t^s)^\alpha (N_t^s)^{1 - \alpha} = M_t f + (M_t - M_t^x) \left(\frac{1}{a} \frac{\chi_t}{1 - \chi_t} \right) + \frac{M_t^e}{\chi_t^e} \left(f^e + \frac{1}{a^e} \frac{\chi_t^e}{1 - \chi_t^e} \right) \quad (\text{A.3.18})$$

$$\frac{r_t^k}{\hat{w}_t^s} = \frac{\alpha}{1 - \alpha} \frac{N_t^s}{\hat{K}_t^s} \quad (\text{A.3.19})$$

Unskilled sector

$$\hat{Y}_t = Z_t M_t^{\frac{1}{\sigma - 1}} (\hat{K}_t^p)^\alpha (N_t^p)^{1 - \alpha} \quad (\text{A.3.20})$$

$$\hat{w}_t^u = (1 - \alpha) \frac{\sigma - 1}{\sigma} Z_t M_t^{\frac{1}{\sigma - 1}} (\hat{K}_t^p)^\alpha (N_t^p)^{-\alpha} / (1 + \zeta r_t^l) \quad (\text{A.3.21})$$

$$r_t^k = \alpha \frac{\sigma - 1}{\sigma} Z_t M_t^{\frac{1}{\sigma - 1}} (\hat{K}_t^p)^{\alpha - 1} (N_t^p)^{1 - \alpha} / (1 + \zeta r_t^l) \quad (\text{A.3.22})$$

Households

$$1 = \text{E}_t \left[\beta \left(g_t^Q \cdot \hat{C}_{t+1} / \hat{C}_t \right)^{-\theta} (1 + r_{t+1}^d) \right] \quad (\text{A.3.23})$$

$$\Lambda_{t,t+1} = \text{E}_t \left[\left(g_t^Q \cdot \hat{C}_{t+1} / \hat{C}_t \right)^{-\theta} \right] \quad (\text{A.3.24})$$

Financial system

$$r_t^l = s p_t + r_t^d \quad (\text{A.3.25})$$

$$r_t^l = r_t^k - d p \quad (\text{A.3.26})$$

Frictional labor markets (notation $w_t^* \equiv w_t(r)$)

$$m_t^u = \sigma_m (u_t^u)^\psi (v_t^u)^{1-\psi} \quad (\text{A.3.27})$$

$$m_t^s = \sigma_m (u_t^s)^\psi (v_t^s)^{1-\psi} \quad (\text{A.3.28})$$

$$n_{t+1}^u = (\rho^u + x_t^u) n_t^u \quad (\text{A.3.29})$$

$$n_{t+1}^s = (\rho^s + x_t^s) n_t^s \quad (\text{A.3.30})$$

$$u_t^u = 1 - n_t^u \quad (\text{A.3.31})$$

$$u_t^s = 1 - n_t^s \quad (\text{A.3.32})$$

$$q_t^u = m_t^u / v_t^u \quad (\text{A.3.33})$$

$$q_t^s = m_t^s / v_t^s \quad (\text{A.3.34})$$

$$p_t^u = m_t^u / u_t^u \quad (\text{A.3.35})$$

$$p_t^s = m_t^s / u_t^s \quad (\text{A.3.36})$$

$$x_t^u = q_t^u v_t^u / n_t^u \quad (\text{A.3.37})$$

$$x_t^s = q_t^s v_t^s / n_t^s \quad (\text{A.3.38})$$

$$\kappa^u x_t^u = \text{E}_t \left[\beta \Lambda_{t,t+1} \left(\hat{w}_{t+1}^u - \hat{w}_t^u + \frac{\kappa^u}{2} (x_{t+1}^u)^2 + \rho^u \kappa^u x_{t+1}^u \right) \right] \quad (\text{A.3.39})$$

$$\kappa^s x_t^s = \text{E}_t \left[\beta \Lambda_{t,t+1} \left(\hat{w}_{t+1}^s - \hat{w}_t^s + \frac{\kappa^s}{2} (x_{t+1}^s)^2 + \rho^s \kappa^s x_{t+1}^s \right) \right] \quad (\text{A.3.40})$$

$$\kappa^u x_t^{u*} = \text{E}_t \left[\beta \Lambda_{t,t+1} \left(\hat{w}_{t+1}^u - \hat{w}_t^{u*} + \frac{\kappa^u}{2} (x_{t+1}^{u*})^2 + \rho^u \kappa^u x_{t+1}^{u*} \right) \right] \quad (\text{A.3.41})$$

$$\kappa^s x_t^{s*} = \text{E}_t \left[\beta \Lambda_{t,t+1} \left(\hat{w}_{t+1}^s - \hat{w}_t^{s*} + \frac{\kappa^s}{2} (x_{t+1}^{s*})^2 + \rho^s \kappa^s x_{t+1}^{s*} \right) \right] \quad (\text{A.3.42})$$

$$\Delta_t^u = 1 + \beta \rho^u \lambda \text{E}_t \left[\Lambda_{t,t+1} g_t^Q \Delta_{t+1}^u \right] \quad (\text{A.3.43})$$

$$\Delta_t^s = 1 + \beta \rho^s \lambda \text{E}_t \left[\Lambda_{t,t+1} g_t^Q \Delta_{t+1}^s \right] \quad (\text{A.3.44})$$

$$\Delta_t^u \hat{w}_t^{u*} = \hat{w}_t^{uo} + \rho^u \lambda \text{E}_t \left[\beta \Lambda_{t,t+1} \Delta_{t+1}^u \hat{w}_{t+1}^{u*} \right] \quad (\text{A.3.45})$$

$$\Delta_t^s \hat{w}_t^{s*} = \hat{w}_t^{so} + \rho^s \lambda \text{E}_t \left[\beta \Lambda_{t,t+1} \Delta_{t+1}^s \hat{w}_{t+1}^{s*} \right] \quad (\text{A.3.46})$$

$$\hat{w}_t^{uf} = \psi \left(\hat{w}_t^u + \frac{\kappa^u}{2} (x_t^u)^2 + p_t^u \kappa^u x_t^u \right) + (1 - \psi) b_t^u \quad (\text{A.3.47})$$

$$\hat{w}_t^{sf} = \psi \left(\hat{w}_t^s + \frac{\kappa^s}{2} (x_t^s)^2 + p_t^s \kappa^s x_t^s \right) + (1 - \psi) b_t^s \quad (\text{A.3.48})$$

$$\begin{aligned} \hat{w}_t^{uo} &= \hat{w}_t^{uf} + \psi \left(\frac{\kappa^u}{2} ((x_t^{u*})^2 - (x_t^u)^2) + p_t^u \kappa^u (x_t^{u*} - x_t^u) \right) \\ &\quad + (1 - \psi) p_t^u \text{E}_t \left[\beta \Lambda_{t,t+1} \lambda \Delta_{t+1}^u g_t^Q (\hat{w}_t^u - \hat{w}_t^{u*}) \right] \end{aligned} \quad (\text{A.3.49})$$

$$\begin{aligned} \hat{w}_t^{so} &= \hat{w}_t^{sf} + \psi \left(\frac{\kappa^s}{2} ((x_t^{s*})^2 - (x_t^s)^2) + p_t^s \kappa^s (x_t^{s*} - x_t^s) \right) \\ &\quad + (1 - \psi) p_t^s \text{E}_t \left[\beta \Lambda_{t,t+1} \lambda \Delta_{t+1}^s g_t^Q (\hat{w}_t^s - \hat{w}_t^{s*}) \right] \end{aligned} \quad (\text{A.3.50})$$

$$\hat{w}_t^u = \lambda \hat{w}_{t-1}^u + (1 - \lambda) \hat{w}_t^{u*} \quad (\text{A.3.51})$$

$$\hat{w}_t^s = \lambda \hat{w}_{t-1}^s + (1 - \lambda) \hat{w}_t^{s*} \quad (\text{A.3.52})$$

$$\hat{b}_t^u = 0.4 \hat{w}_{ss}^u \quad (\text{A.3.53})$$

$$\hat{b}_t^s = 0.4 \hat{w}_{ss}^s \quad (\text{A.3.54})$$

Market clearing

$$\hat{Y}_t = \hat{C}_t + \hat{I}_t + \kappa^u (x_t^u)^2 N_t^p + \kappa^s (x_t^s)^2 N_t^s \quad (\text{A.3.55})$$

$$g_t^Q \hat{K}_{t+1} = (1 - dp) \hat{K}_t + \hat{I}_t \quad (\text{A.3.56})$$

$$\hat{K}_t = \hat{K}_t^p + \hat{K}_t^s \quad (\text{A.3.57})$$

$$N_t^p = (1 - s) n_t^u \quad (\text{A.3.58})$$

$$N_t^s = s n_t^s \quad (\text{A.3.59})$$

Shocks

$$\log Z_t = \rho_Z \log Z_{t-1} + \varepsilon_{Z,t} \quad (\text{A.3.60})$$

$$\log sp_t = (1 - \rho_{sp}) \log sp_{ss} + \rho_{sp} \log sp_{t-1} + \varepsilon_{sp,t} \quad (\text{A.3.61})$$

Welfare

$$U_t = \frac{(\hat{C}_t Q_t)^{1-\theta}}{1-\theta} + \beta \mathbb{E}_t [U_{t+1}] \quad (\text{A.3.62})$$

A.3.2 Additional derivations

A.3.2.1 Solutions of cost minimization problems

Intermediate goods production sector

$$\begin{aligned} \min \quad & tc_t^p(i) = (1 + \zeta r_t^l) (\tilde{w}_t^u n_t^p(i) + r_t k_t^p(i)) \\ \text{subject to} \quad & y_t(i) = Z_t k_t^p(i)^\alpha [q_t(i) n_t^p(i)]^{1-\alpha} \end{aligned}$$

FOCs

$$\begin{aligned} n_t(i) &: (1 + \zeta r_t^l) \tilde{w}_t^u = \lambda^p (1 - \alpha) Z_t k_t^p(i)^\alpha q_t(i)^{1-\alpha} n_t^p(i)^{-\alpha} \\ k_t(i) &: (1 + \zeta r_t^l) r_t = \lambda^p \alpha Z_t k_t^p(i)^{\alpha-1} q_t(i)^{1-\alpha} n_t^p(i)^{1-\alpha} \end{aligned}$$

Divide

$$\begin{aligned} \frac{\tilde{w}_t^u}{r_t} &= \frac{1 - \alpha}{\alpha} \frac{k_t^p(i)}{n_t^p(i)} \\ k_t^p(i) &= \frac{\alpha}{1 - \alpha} \frac{\tilde{w}_t^u}{r_t} n_t^p(i) \\ n_t^p(i) &= \frac{1 - \alpha}{\alpha} \frac{r_t}{\tilde{w}_t^u} k_t^p(i) \end{aligned}$$

Production function

$$\begin{aligned} y_t(i) &= Z_t k_t^p(i)^\alpha [q_t(i) n_t^p(i)]^{1-\alpha} = Z_t k_t^p(i)^\alpha \left[q_t(i) \frac{1 - \alpha}{\alpha} \frac{r_t}{\tilde{w}_t^u} k_t^p(i) \right]^{1-\alpha} \\ &= Z_t k_t^p(i) \left[q_t(i) \frac{1 - \alpha}{\alpha} \frac{r_t}{\tilde{w}_t^u} \right]^{1-\alpha} \\ k_t^p(i) &= \frac{y_t(i)}{Z_t} \left[q_t(i) \frac{1 - \alpha}{\alpha} \frac{r_t}{\tilde{w}_t^u} \right]^{\alpha-1} \end{aligned}$$

Total cost

$$\begin{aligned} tc_t^p(i) &= (1 + \zeta r_t^l) (\tilde{w}_t^u n_t^p(i) + r_t k_t^p(i)) = (1 + \zeta r_t^l) \left(\tilde{w}_t^u \frac{1 - \alpha}{\alpha} \frac{r_t}{\tilde{w}_t^u} k_t^p(i) + r_t k_t^p(i) \right) \\ &= (1 + \zeta r_t^l) \left(\frac{1 - \alpha}{\alpha} + 1 \right) r_t k_t^p(i) = (1 + \zeta r_t^l) \frac{r_t}{\alpha} k_t^p(i) \\ &= (1 + \zeta r_t^l) \frac{r_t}{\alpha} \frac{y_t(i)}{Z_t} \left[q_t(i) \frac{1 - \alpha}{\alpha} \frac{r_t}{\tilde{w}_t^u} \right]^{\alpha-1} = (1 + \zeta r_t^l) \frac{y_t(i)}{Z_t} \left(\frac{r_t}{\alpha} \right)^\alpha \left(\frac{\tilde{w}_t^u / q_t(i)}{1 - \alpha} \right)^{1-\alpha} \end{aligned}$$

Real marginal cost

$$mc_t^p(i) = \frac{(1 + \zeta r_t^l)}{Z_t} \left(\frac{r_t}{\alpha} \right)^\alpha \left(\frac{\tilde{w}_t^u / q_t(i)}{1 - \alpha} \right)^{1-\alpha}$$

Research and development sector

$$\begin{aligned} \min \quad & tc_t^x(i) = (1 + \zeta r_t^l) (\tilde{w}_t^s n_t^x(i) + r_t k_t^x(i)) \\ \text{subject to} \quad & x_t(i) = \frac{k_t^x(i)^\alpha [Q_t n_t^x(i)]^{1-\alpha}}{Q_t \phi_t(i)} \end{aligned}$$

FOCs

$$\begin{aligned} n_t^x(i) \quad &: \quad (1 + \zeta r_t^l) \tilde{w}_t^s = \lambda (1 - \alpha) \frac{Z_t k_t^x(i)^\alpha Q_t^{1-\alpha} n_t^x(i)^{-\alpha}}{Q_t \phi_t(i)} \\ k_t^x(i) \quad &: \quad (1 + \zeta r_t^l) r_t = \lambda \alpha \frac{Z_t k_t^x(i)^{\alpha-1} Q_t^{1-\alpha} n_t^x(i)^{1-\alpha}}{Q_t \phi_t(i)} \end{aligned}$$

Divide

$$\begin{aligned} \frac{\tilde{w}_t^s}{r_t} &= \frac{1 - \alpha}{\alpha} \frac{k_t^x(i)}{n_t^x(i)} \\ k_t^x(i) &= \frac{\alpha}{1 - \alpha} \frac{\tilde{w}_t^s}{r_t} n_t^x(i) \\ n_t^x(i) &= \frac{1 - \alpha}{\alpha} \frac{r_t}{\tilde{w}_t^s} k_t^x(i) \end{aligned}$$

R&D production function

$$\begin{aligned} x_t(i) &= \frac{k_t^x(i)^\alpha [Q_t n_t^x(i)]^{1-\alpha}}{Q_t \phi_t(i)} = Q_t^{-\alpha} k_t^x(i) \left(\frac{1 - \alpha}{\alpha} \frac{r_t}{\tilde{w}_t^s} \right)^{1-\alpha} / \phi_t(i) \\ k_t^x(i) &= x_t(i) Q_t^\alpha \left(\frac{1 - \alpha}{\alpha} \frac{r_t}{\tilde{w}_t^s} \right)^{\alpha-1} \phi_t(i) \end{aligned}$$

Total cost

$$\begin{aligned} tc_t^x(i) &= (1 + \zeta r_t^l) \frac{r_t}{\alpha} k_t^x(i) = (1 + \zeta r_t^l) \frac{r_t}{\alpha} x_t(i) Q_t^\alpha \left(\frac{1 - \alpha}{\alpha} \frac{r_t}{\tilde{w}_t^s} \right)^{\alpha-1} \phi_t(i) \\ &= (1 + \zeta r_t^l) x_t(i) Q_t^\alpha \left(\frac{r_t}{\alpha} \right)^\alpha \left(\frac{\tilde{w}_t^s}{1 - \alpha} \right)^{1-\alpha} \phi_t(i) \end{aligned}$$

Real marginal cost

$$mc_t^x(i) = (1 + \zeta r_t^l) Q_t^\alpha \left(\frac{r_t}{\alpha} \right)^\alpha \left(\frac{\tilde{w}_t^s}{1 - \alpha} \right)^{1-\alpha} \phi_t(i) \equiv \bar{mc}_t^x \phi_t(i)$$

Total cost as function of desired innovative success probability

$$\begin{aligned} \chi_t(i) &= \frac{ax_t(i)}{1 + ax_t(i)} \\ x_t(i) &= \frac{1}{a} \frac{\chi_t(i)}{1 - \chi_t(i)} \\ tc_t^x(i) &= (1 + \zeta r_t^l) \frac{\bar{mc}_t^x}{a} \frac{\chi_t(i)}{1 - \chi_t(i)} \phi_t(i) \end{aligned}$$

A.3.2.2 Aggregate production function

Relative inputs

$$\begin{aligned}
\frac{y_t(i)}{y_t(j)} &= \frac{Y_t p_t(i)^{-\sigma}}{Y_t p_t(j)^{-\sigma}} = \left[\frac{\frac{\sigma}{\sigma-1} \frac{(1+\zeta r_t^l)}{Z_t} \left(\frac{r_t}{\alpha}\right)^\alpha \left(\frac{\bar{w}_t^u/q_t(i)}{1-\alpha}\right)^{1-\alpha}}{\frac{\sigma}{\sigma-1} \frac{(1+\zeta r_t^l)}{Z_t} \left(\frac{r_t}{\alpha}\right)^\alpha \left(\frac{\bar{w}_t^u/q_t(j)}{1-\alpha}\right)^{1-\alpha}} \right]^{-\sigma} = \left(\frac{q_t(i)^{\alpha-1}}{q_t(j)^{\alpha-1}} \right)^{-\sigma} = \left(\frac{q_t(i)^{1-\alpha}}{q_t(j)^{1-\alpha}} \right)^\sigma \\
\frac{y_t(i)}{y_t(j)} &= \frac{Z_t k_t^p(i) \left[q_t(i)^{\frac{1-\alpha}{\alpha}} \frac{r_t}{\bar{w}_t^u} \right]^{1-\alpha}}{Z_t k_t^p(j) \left[q_t(j)^{\frac{1-\alpha}{\alpha}} \frac{r_t}{\bar{w}_t^u} \right]^{1-\alpha}} \\
\frac{k_t^p(i) q_t(i)^{1-\alpha}}{k_t^p(j) q_t(j)^{1-\alpha}} &= \left(\frac{q_t(i)^{1-\alpha}}{q_t(j)^{1-\alpha}} \right)^\sigma \\
\frac{k_t^p(i)}{k_t^p(j)} &= \left(\frac{q_t(i)}{q_t(j)} \right)^{(1-\alpha)(\sigma-1)} \\
k_t^p(i) &= \left(\frac{q_t(i)}{q_t(j)} \right)^{(1-\alpha)(\sigma-1)} k_t^p(j) \\
k_t^p(i) &= \left(\frac{q_t(i)}{Q_t} \right)^{(1-\alpha)(\sigma-1)} \bar{k}_t^p \\
n_t^p(i) &= \left(\frac{q_t(i)}{Q_t} \right)^{(1-\alpha)(\sigma-1)} \bar{n}_t^p
\end{aligned}$$

where $\bar{k}_t^p \equiv K_t^p/M_t$ and $\bar{n}_t^p \equiv N_t^p/M_t$.

Final goods output

$$\begin{aligned}
Y_t &= \left[\int_0^{M_t} y_t(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}} = \left[M_t \int_0^\infty y_t(q)^{\frac{\sigma-1}{\sigma}} \mu_t(q) dq \right]^{\frac{\sigma}{\sigma-1}} \\
&= M_t^{\frac{\sigma}{\sigma-1}} \left[\int_0^\infty \left[Z_t k_t^p(q)^\alpha q^{1-\alpha} n_t^p(q)^{1-\alpha} \right]^{\frac{\sigma-1}{\sigma}} \mu_t(q) dq \right]^{\frac{\sigma}{\sigma-1}} \\
&= M_t^{\frac{\sigma}{\sigma-1}} Z_t \left[\int_0^\infty \left[\left(\frac{q}{Q_t} \right)^{(1-\alpha)(\sigma-1)} (\bar{k}_t^p)^\alpha (\bar{n}_t^p)^{1-\alpha} q^{1-\alpha} \right]^{\frac{\sigma-1}{\sigma}} \mu_t(q) dq \right]^{\frac{\sigma}{\sigma-1}} \\
&= M_t^{\frac{\sigma}{\sigma-1}} Z_t (\bar{k}_t^p)^\alpha (\bar{n}_t^p)^{1-\alpha} Q_t^{(1-\alpha)(1-\sigma)} \left[\int_0^\infty \left[(q^{1-\alpha})^\sigma \right]^{\frac{\sigma-1}{\sigma}} \mu_t(q) dq \right]^{\frac{\sigma}{\sigma-1}} \\
&= M_t^{\frac{\sigma}{\sigma-1}-1} Z_t (K_t^p)^\alpha (N_t^p)^{1-\alpha} Q_t^{(1-\alpha)(1-\sigma)} \left[\int_0^\infty (q^{1-\alpha})^{\sigma-1} \mu_t(q) dq \right]^{\frac{1}{\sigma-1}}^\sigma \\
&= M_t^{\frac{1}{\sigma-1}} Z_t (K_t^p)^\alpha (N_t^p)^{1-\alpha} Q_t^{(1-\alpha)(1-\sigma)} (Q_t^{1-\alpha})^\sigma \\
&= M_t^{\frac{1}{\sigma-1}} Z_t (K_t^p)^\alpha (Q_t N_t^p)^{1-\alpha}
\end{aligned}$$

A.3.2.3 Real profit function

Real operating profit

$$\begin{aligned}
\pi_t^o(i) &= p_t(i) y_t(i) - mc_t^p(i) y_t(i) - f_t = p_t(i) y_t(i) - p_t(i) \frac{\sigma-1}{\sigma} y_t(i) - f_t \\
&= \left(1 - \frac{\sigma-1}{\sigma}\right) Y_t p_t(i)^{1-\sigma} - f_t = \frac{1}{\sigma} Y_t \left[\frac{\sigma}{\sigma-1} \frac{(1+\zeta r_t^l)}{Z_t} \left(\frac{r_t}{\alpha}\right)^\alpha \left(\frac{\tilde{w}_t^u/q_t(i)}{1-\alpha}\right)^{1-\alpha} \right]^{1-\sigma} - f_t \\
&= \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} Y_t Z_t^{\sigma-1} \left[(1+\zeta r_t^l) \left(\frac{r_t}{\alpha}\right)^\alpha \left(\frac{\tilde{w}_t^u/q_t(i)}{1-\alpha}\right)^{1-\alpha} \right]^{1-\sigma} - f_t
\end{aligned}$$

Price index (where $R_t \equiv P_t r_t$ and $W_t^u \equiv P_t \tilde{w}_t^u$)

$$\begin{aligned}
P_t &= \left[\int_0^{M_t} P_t(i)^{1-\sigma} di \right]^{\frac{1}{1-\sigma}} = \left[M_t \int_0^\infty P_t(q)^{1-\sigma} \mu_t(q) dq \right]^{\frac{1}{1-\sigma}} \\
&= M_t^{\frac{1}{1-\sigma}} \left[\int_0^\infty \left[\frac{\sigma}{\sigma-1} \frac{(1+\zeta r_t^l)}{Z_t} \left(\frac{R_t}{\alpha}\right)^\alpha \left(\frac{W_t^u/q}{1-\alpha}\right)^{1-\alpha} \right]^{1-\sigma} \mu_t(q) dq \right]^{\frac{1}{1-\sigma}} \\
&= \frac{\sigma}{\sigma-1} M_t^{\frac{1}{1-\sigma}} \frac{(1+\zeta r_t^l)}{Z_t} \left(\frac{R_t}{\alpha}\right)^\alpha \left(\frac{W_t^u}{1-\alpha}\right)^{1-\alpha} \left[\int_0^\infty (q^{\alpha-1})^{1-\sigma} \mu_t(q) dq \right]^{\frac{1}{1-\sigma}} \\
&= \frac{\sigma}{\sigma-1} M_t^{\frac{1}{1-\sigma}} \frac{(1+\zeta r_t^l)}{Z_t} \left(\frac{R_t}{\alpha}\right)^\alpha \left(\frac{W_t^u}{1-\alpha}\right)^{1-\alpha} \left[\int_0^\infty (q^{1-\alpha})^{\sigma-1} \mu_t(q) dq \right]^{\frac{1}{\sigma-1}}]^{-1} \\
&= \frac{\sigma}{\sigma-1} M_t^{\frac{1}{1-\sigma}} \frac{(1+\zeta r_t^l)}{Z_t} \left(\frac{R_t}{\alpha}\right)^\alpha \left(\frac{W_t^u}{1-\alpha}\right)^{1-\alpha} (Q_t^{1-\alpha})^{-1} \\
&= \frac{\sigma}{\sigma-1} M_t^{\frac{1}{1-\sigma}} \frac{(1+\zeta r_t^l)}{Z_t} \left(\frac{R_t}{\alpha}\right)^\alpha \left(\frac{W_t^u/Q_t}{1-\alpha}\right)^{1-\alpha}
\end{aligned}$$

Real input cost index

$$\begin{aligned}
(1+\zeta r_t^l) \left(\frac{R_t}{\alpha}\right)^\alpha \left(\frac{W_t^u/Q_t}{1-\alpha}\right)^{1-\alpha} &= \frac{\sigma-1}{\sigma} P_t M_t^{\frac{1}{\sigma-1}} Z_t \\
(1+\zeta r_t^l) \left(\frac{r_t}{\alpha}\right)^\alpha \left(\frac{\tilde{w}_t^u}{1-\alpha}\right)^{1-\alpha} &= \frac{\sigma-1}{\sigma} M_t^{\frac{1}{\sigma-1}} Z_t Q_t^{1-\alpha}
\end{aligned}$$

Real operating profit

$$\begin{aligned}
\pi_t^o(i) &= \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} Y_t Z_t^{\sigma-1} \left[(1+\zeta r_t^l) \left(\frac{r_t}{\alpha}\right)^\alpha \left(\frac{w_t/q_t(i)}{1-\alpha}\right)^{1-\alpha} \right]^{1-\sigma} - f_t \\
&= \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} Y_t Z_t^{\sigma-1} \left[\frac{\sigma-1}{\sigma} M_t^{\frac{1}{\sigma-1}} Z_t Q_t^{1-\alpha} q_t(i)^{\alpha-1} \right]^{1-\sigma} - f_t \\
&= \frac{Y_t}{\sigma M_t} \left[\left(\frac{q_t(i)}{Q_t}\right)^{1-\alpha} \right]^{\sigma-1} - f_t \\
&= \frac{Y_t}{\sigma M_t} \phi_t(i) - f_t
\end{aligned}$$

Real profit

$$\begin{aligned}
\pi_t(i) &= \pi_t^o(i) - \left(1 + \zeta r_t^l\right) \frac{\bar{m}c_t^x}{a} \frac{\chi_t(i)}{1 - \chi_t(i)} \phi_t(i) \\
&= \left(\frac{Y_t}{\sigma M_t} - \left(1 + \zeta r_t^l\right) \frac{\bar{m}c_t^x}{a} \frac{\chi_t(i)}{1 - \chi_t(i)} \right) \phi_t(i) - f_t \\
&= \left(\frac{Y_t}{\sigma M_t} - \left(1 + \zeta r_t^l\right) \frac{\bar{m}c_t^x}{a} \frac{\chi_t(i)}{1 - \chi_t(i)} \right) \phi_t(i) - \left(1 + \zeta r_t^l\right) \bar{m}c_t^x f \\
&= Y_t \left[\left(\frac{1}{\sigma M_t} - \left(1 + \zeta r_t^l\right) \frac{\omega_t}{a} \frac{\chi_t(i)}{1 - \chi_t(i)} \right) \phi_t(i) - \left(1 + \zeta r_t^l\right) \omega_t f \right]
\end{aligned}$$

A.3.3 Additional tables and figures

Table A.2: Chapter 4: Prior distributions of parameters

Parameter	Description	Distribution shape	Mean	Std. dev.
λ	Calvo wage contract prob.	Uniform $[0, 1]$	0.5	0.289
ρ_Z	Autocorr. of prod. process	Beta	0.7	0.175
σ_Z	Std. dev. of prod. shock	Inverse Gamma	0.01	∞
ρ_{sp}	Autocorr. of spread process	Beta	0.7	0.175
σ_{sp}	Std. dev. of spread shock	Inverse Gamma	0.1	∞

Figure A.1: Chapter 4: Results of the maximum likelihood estimation step

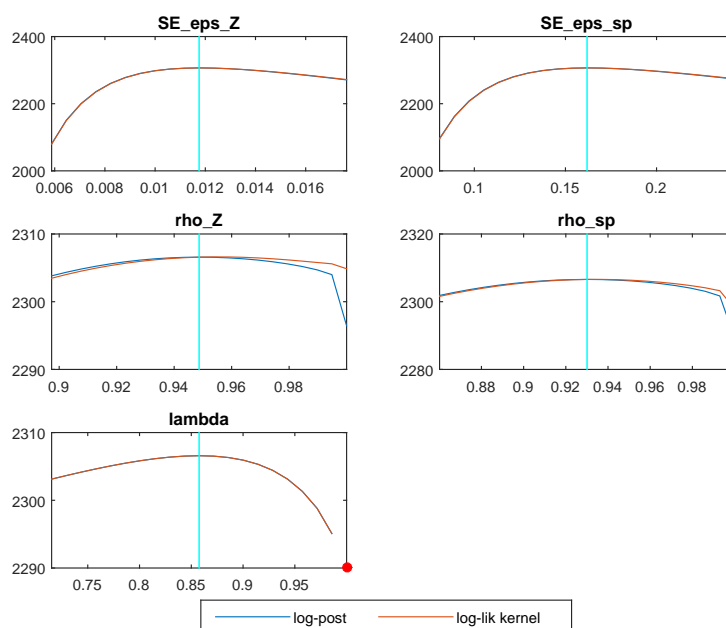


Figure A.2: Chapter 4: Univariate and multivariate convergence of parameters

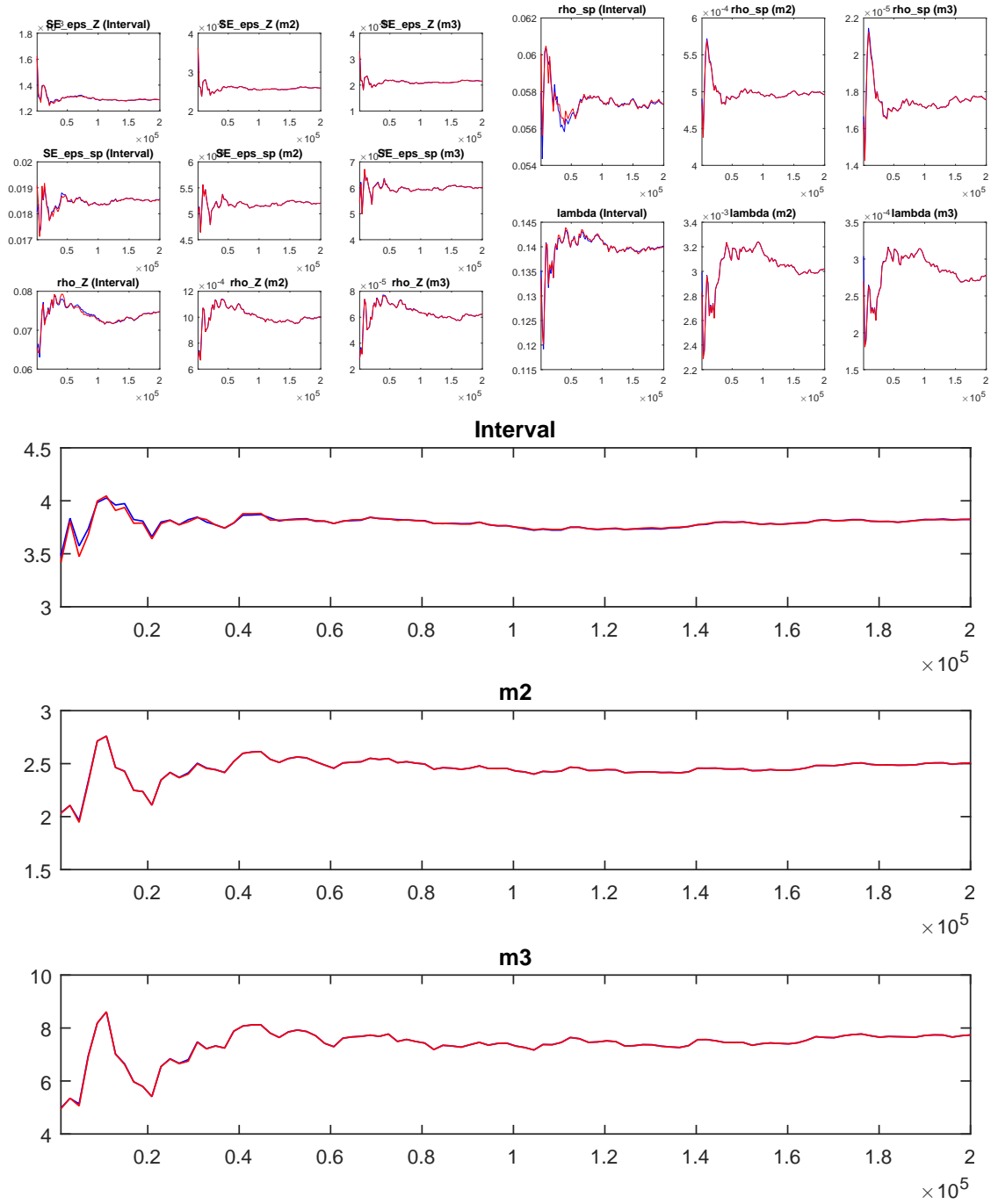


Figure A.3: Chapter 4: Bayesian impulse response functions to productivity shock

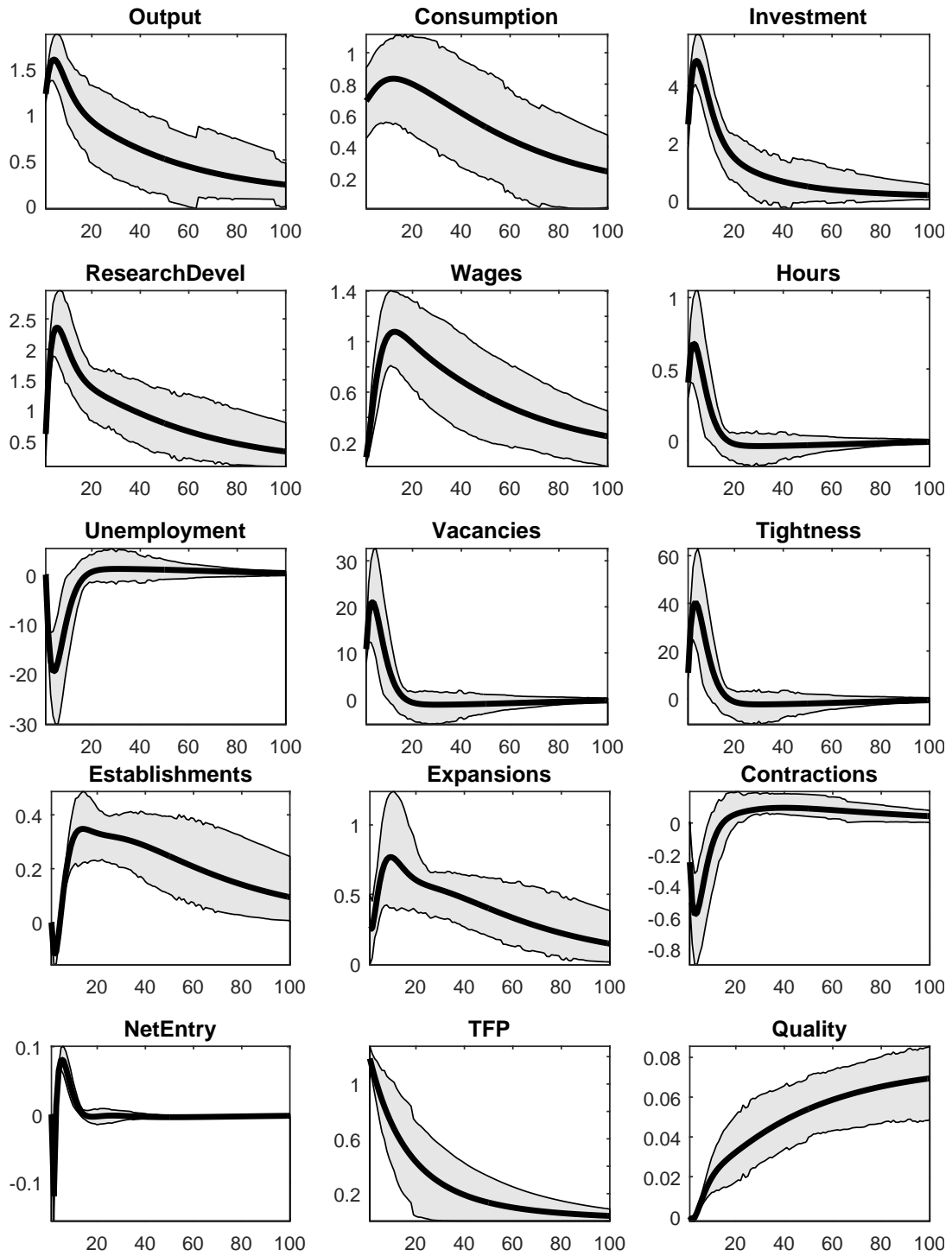
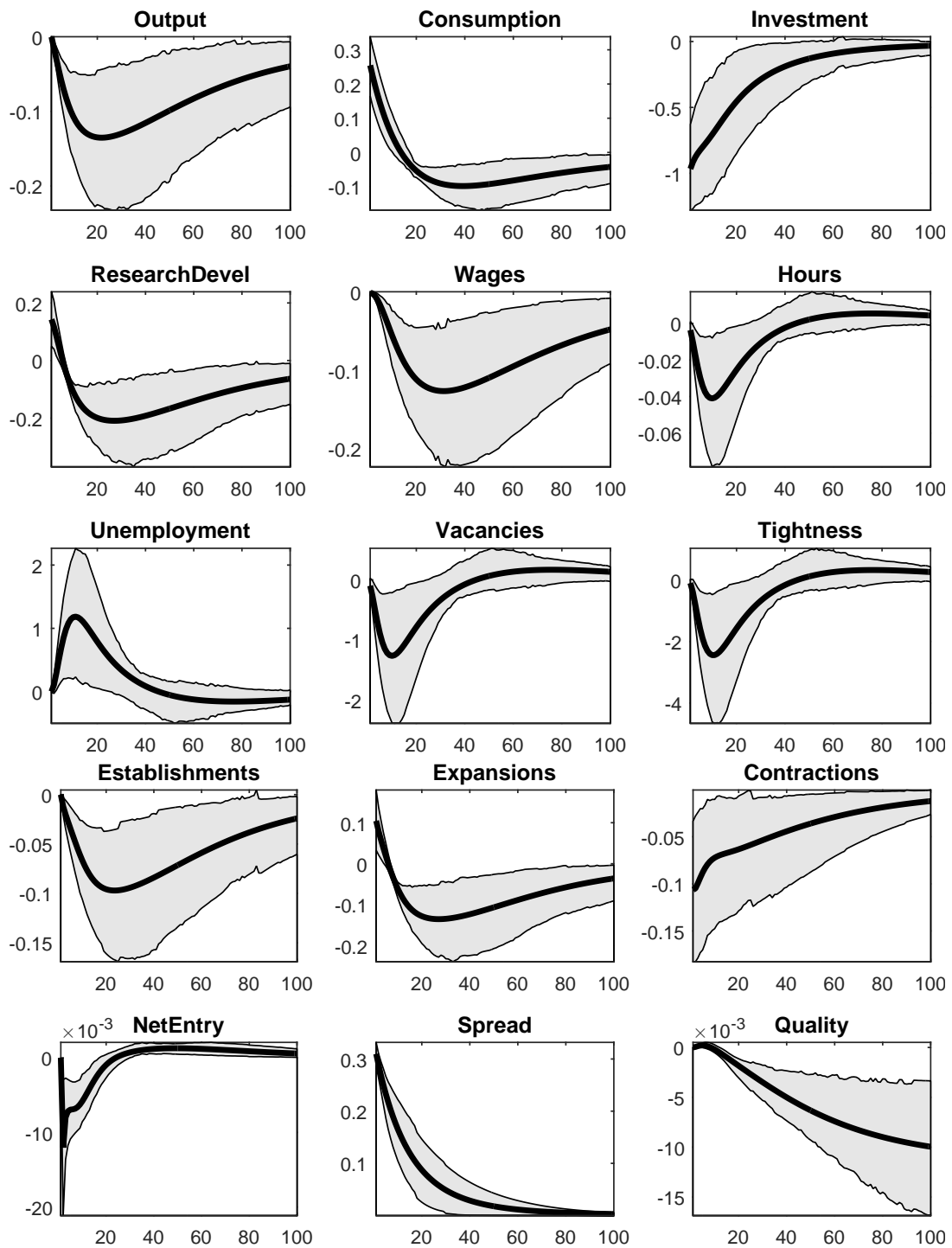


Figure A.4: Chapter 4: Bayesian impulse response functions to interest rate spread shock



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